EFFECT OF VARIATION OF PARAMETERS ON INDUCTIVE STARTING OF DIRECT CURRENT MOTORS

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PREFACE

The work of this thesis was performed at the United States Naval Postgraduate School, Monterey, California, during the period November 1952 to June 1953.

The topic was suggested in Enclosure (2) to BuShips Ltr. Ser. 258-914. This enclosure was a list of proposed thesis subjects contained in a letter to the Postgraduate School from the Chief of the Bureau of Ships, Navy Department, Washington, D.C.

The authors wish to acknowledge the invaluable aid which was extended at all times during the preparation of this thesis, by Professor Charles V. O. Terwilliger, and Professor Orval H. Polk, both of the Electrical Engineering Department of the Postgraduate School.

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TABLE OF SYMBOLS AND ABBREVIATIONS

E	- Direct Current Line Voltage (volts)
E_a	- Generated Back E.M.F. of the Motor (volts)
E.M.F.	- Electromotive Force (volts)
i	- Instantaneous Motor Armature Current (amps)
i_o	- Current at which the Motor Starts (initial current)(amps)
I	- Steady-state Motor Armature Current (amps)
J	- Inertia of all Rotating Parts (slug-ft^2)
K_g	- Generated Back E.M.F. Constant of the Motor(volts/radian/second)
K_t	- Developed Torque Constant of the Motor (lbs-ft/amp)
L	- Inductance External to Motor (henrys)
L_a	- Inductance of Motor Armature (henrys)
L_t	- Total Inductance of Armature Circuit ($L + L_a$)(henrys)
R	- Resistance External to Motor (ohms)
R_a	- Resistance of Motor Armature (ohms)
R_t	- Total Resistance of Armature Circuit ($R + R_a$)(ohms)
t	- Time after Motor Starts (seconds)
T	- Developed Torque of the Motor (lbs-ft)
T_c	- Load Torque which is constant (lbs-ft)
T_s	- Load Torque Constant for Load Torque which is proportional to speed (lbs-ft/rad/sec)
T_t	- Total Load Torque ($T_c + T_s\omega$)(lbs-ft)
ω	- Instantaneous Motor Speed (radians/second)

SUMMARY

The object of this thesis was to investigate the effect of varying certain parameters on the inductive starting of direct current motors. These parameters were the external inductance of the armature circuit, the inertia of all rotating parts and the amount and type of load.

The investigation was a theoretical study made by solving the equations for motor current and speed as a function of time on an electronic analog computer. First, however, the validity of the theoretical solutions were checked by comparing certain runs which were made both on the analog computer and on a 7.5 horsepower motor in the Electrical Engineering Laboratory at the U. S. Naval Postgraduate School. These runs checked quite closely. Thirty eight additional solutions for the 7.5 horsepower motor were then obtained using the analog computer varying the above parameters.

It was found that with certain combinations of the parameters, severe oscillations of current and speed exist for as long as one minute after starting. However, these oscillations can be limited to reasonable values by the proper choice of circuit parameters. With some combinations of circuit parameters and load conditions the oscillations are completely eliminated and critical or over-damping is obtained.

CHAPTER I

STARTING DIRECT CURRENT MOTORS

It is necessary to provide all but the smallest direct current motors with a starting device to limit the armature current during the starting period. If the motor were connected directly across the line, the current at the instant of closing the switch would be limited only by the resistance (and very slight inductance) of the armature circuit, which is insufficient to limit it to a reasonable value. The starting device must function until the motor speed is high enough for the back e.m.f. to limit the current.

In order for the motor to accelerate, the developed torque must exceed the resisting torque of the load including rotational losses. The developed torque is proportional to the product of armature current and field flux. Assuming that the flux remains constant, as is nearly the case for a shunt motor, the torque is proportional to armature current. Therefore the current during the starting period must be sufficiently high to produce acceleration, yet must be limited to a reasonable value.

Direct current motor starting devices in common use today are all of the resistance type. They limit the armature current by providing external resistance in series with the armature and by removing it from the circuit as the motor speeds up. To provide smooth acceleration, it is necessary to remove the resistance gradually and continuously; but, as this is impractical, it is usually removed in two or more steps. This may be done either manually or by means of automatic starters

controlled by back e.m.f., armature current, or time.

Manually operated resistance starters have the disadvantage that the rate at which the resistance is removed from the circuit is left to the judgement of the operator. If the operation is too fast, excessive current results which may in itself damage the motor, and the resulting rapid acceleration may damage either the motor or its connected load. If the operation is too slow, the resistors may overheat and burn out.

Automatic starters attempt to overcome these difficulties by removing the human factor. However, variations in line voltage or motor load may cause troubles similar to those described above.

In order to provide better starting performance under all conditions, resistance type starters become more and more complicated, consisting of many relays and contacts. The initial cost is high, the maintenance is expensive, and as they become more complicated, they are more susceptable to malfunction, resulting in damage to starter elements or to the motor itself.

To overcome some of the disadvantages of resistance type starters, a device may be used consisting merely of an inductance in series with the armature. The chief advantage of such a device is its extreme simplicity and resultant freedom from operating difficulties. A second advantage which might be important in some applications, is the possibility of very smooth starting due to the continuous nature of the current variation.

An investigation of such a device was conducted by Minner and

Armstrong (1) in which they calculated the effect of using a 5 henry inductance for starting a 7.5 horsepower motor with a constant torque load. Their results indicated severe hunting and large current oscillations existed during the starting period. The purpose of this investigation was to determine the effect of the various circuit parameters on inductive starting characteristics and to determine a set of circuit parameters which will give satisfactory starting performance for a similar machine.

CHAPTER II

MATHMATICAL DEVELOPMENT OF INDUCTIVE STARTING

The development of the motor starting equations for current and speed as proposed by Minner and Armstrong (1) was used and is restated here with the addition of a term representing that portion of load torque which is proportional to speed. The electrical circuit is shown in Figure 1.

The voltage equation for the circuit is

$$E = L_t \frac{di}{dt} + R_t i + E_a . \tag{1}$$

The back e.m.f. is proportional to motor speed,

$$E_a = K_g \omega , \tag{2}$$

and the voltage equation can be written

$$E = L_t \frac{di}{dt} + R_t i + K_g \omega . \tag{3}$$

The developed torque of the motor is proportional to armature current, assuming armature reaction effects are negligible,

$$T = K_t i , \tag{4}$$

and is equal to the sum of the acceleration and load torques,

$$T = J \frac{d\omega}{dt} + T_t . \tag{5}$$

Solving equations (4) and (5) for armature current,

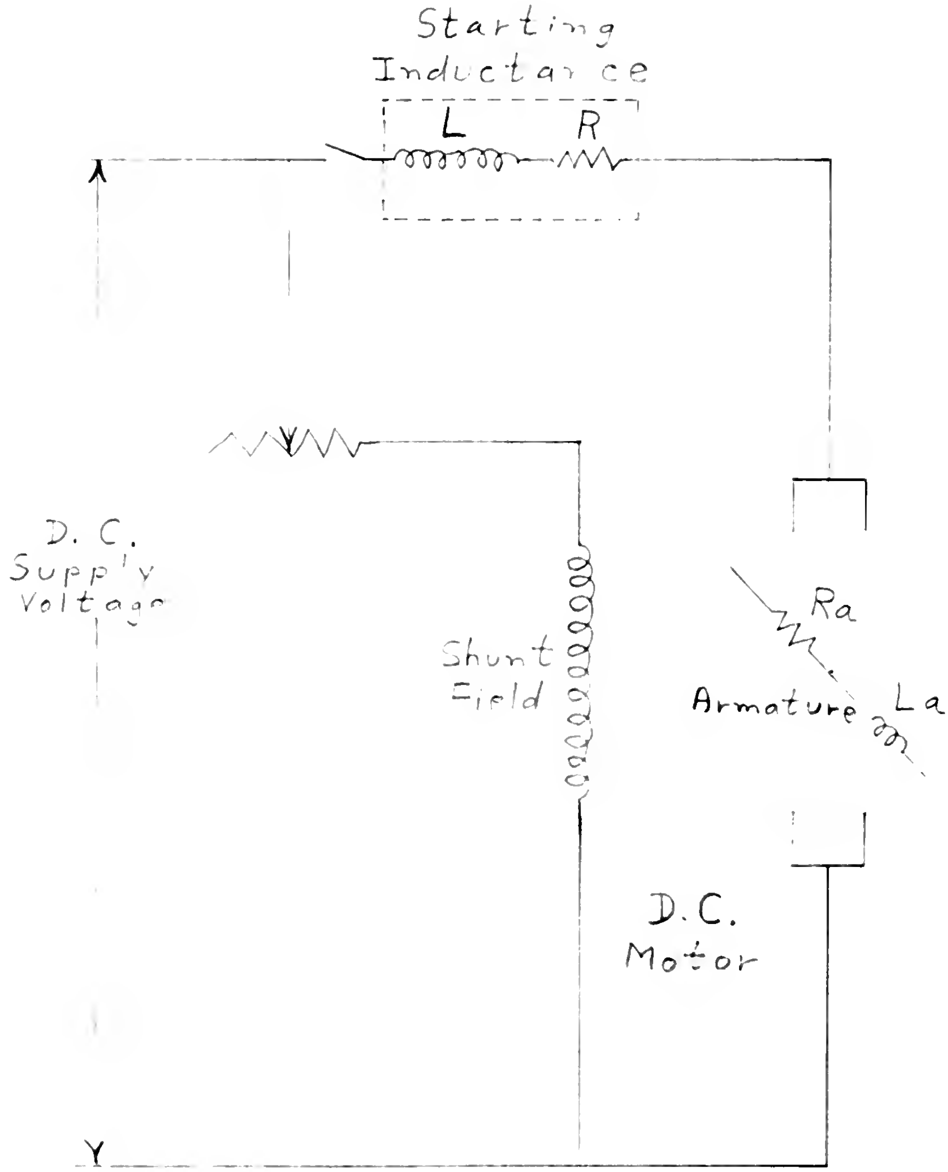
$$i = \frac{J}{K_t} \frac{d\omega}{dt} + \frac{T_t}{K_t} . \tag{6}$$

For this analysis the load torque is assumed to consist of a constant torque and a torque which is proportional to speed,

$$T_t = T_c + T_s \omega . \tag{7}$$

The current now becomes

$$i = \frac{J}{K_t} \frac{d\omega}{dt} + \frac{T_s \omega}{K_t} + \frac{T_c}{K_t} , \tag{8}$$



Basic Electrical Circuit for Inductance Starting

Figure 1

and

$$\frac{di}{dt} = \frac{J}{K_t}\frac{d^2\omega}{dt^2} + \frac{T_s}{K_t}\frac{d\omega}{dt} . \tag{9}$$

Substituting equations (8) and (9) for i and $\frac{di}{dt}$ in equation (3),

$$E = \frac{L_t J}{K_t}\frac{d^2\omega}{dt^2} + \frac{L_t T_s}{K_t}\frac{d\omega}{dt} + \frac{R_t J}{K_t}\frac{d\omega}{dt} + \frac{R_t T_s}{K_t}\omega + \frac{R_t T_c}{K_t} + K_g\omega . \tag{10}$$

Multiplying by $\frac{K_t}{L_t J}$ and rearranging,

$$\frac{d^2\omega}{dt^2} + \left(\frac{T_s}{J} + \frac{R_t}{L_t}\right)\frac{d\omega}{dt} + \left(\frac{R_t T_s}{L_t J} + \frac{K_g K_t}{L_t J}\right)\omega + \left(\frac{R_t T_c - K_t E}{L_t J}\right) = 0. \tag{11}$$

At zero time

$$\omega = 0, \tag{12}$$

$$i = i_o, \tag{13}$$

where i_o is the value of current necessary to overcome static friction. Substituting these initial conditions in equation (8) and solving for $\frac{d\omega}{dt}$,

$$\frac{d\omega}{dt} = \frac{i_o K_t - T_c}{J} . \tag{14}$$

Equations (12) and (14) are the initial conditions for equation (11) which can be solved for motor speed at any time after starting. Substituting the proper values of ω and $\frac{d\omega}{dt}$ in equation (8) will give the corresponding armature current.

CHAPTER III

PROCEDURE AND RESULTS

1. General

The basic objective of this thesis was to determine the effect of variation of parameters on the inductive starting characteristics of direct current motors. It was beyond the scope of the thesis to attempt to study the effect of variation of all circuit and motor parameters. Those which were selected as variable and the reason for their selection, as well as an explanation for the fixed values assigned to the other parameters are discussed in section 2 of this chapter.

Two methods of approach were considered in the study of the effect of the variation of the selected parameters: first, theoretical studies, and second, actual experimental work on a direct current motor in the laboratory. Since an electronic analog computer was available, and relatively wide variation of parameters was desired, the theoretical approach was deemed the more practical. First, however, it was desired to verify the validity of the theoretical study by checking some comparable computer solutions and laboratory runs.

A suitable motor was selected and some starting runs were made with a starting inductance. The motor and circuit constants were determined and a theoretical solution was then made on the analog computer, using these constants. The actual laboratory runs checked quite closely with the computer solutions. The investigation was then extended on the analog computer. The laboratory runs are discussed in

section 3, while the analog computer solutions are described in section 4. Section 5 gives the procedures for the extension of the theoretical study with a summation of the results.

2. Selection of Variable and Fixed Parameters

For the purposes of this thesis, it was decided to assume that the investigation of inductance starting was being conducted with a motor which was already in service. All motor constants were obtained as described in Appendix A. These constants were then assumed to be fixed with the one exception of the inertia of all rotating parts. The total inertia can be effectively increased in practice by the use of flywheels. The shunt field current was kept constant throughout all runs.

Two basic types of loading were considered. One was a load which required a constant torque. The other was a load which required a torque proportional to speed. No combination of these types of load was investigated. However, full and fractional loads of both types were considered.

A constant which was arbitrarily selected was the initial current required to overcome static friction. For the constant torque loads, a value of starting current of 1.5 times steady-state current was selected. For those loads which required a torque proportional to speed, an initial current of 20 amperes was used throughout.

The inductance used in the laboratory and its characteristics are discussed in Appendix C. Average values of its inductance and resistance

were used in the verification runs of sections 3 and 4. For the extended investigation, it was seen that the value of the inductance was one of the most important control variables. An inductance with very little resistance was desired, but was hardly practical as explained below.

The resistance of the inductance should be kept as low as possible in order to minimize copper loss and speed regulation if the inductance is to remain in the circuit; and to minimize the secondary transient effect if the inductance is to be removed from the circuit after the starting period. A value of 0.5 ohms was selected as one which would be practically attainable consistent with the size of inductance required. This value of resistance was held constant for all runs with the exception of the runs used for comparison of theoretical and laboratory results.

The variable parameters studied were:

a. Motor inertia (J).

b. Value of starting inductance (L).

c. Type and amount of motor loads (T_c and T_s).

3. <u>Laboratory Starting of a Direct Current Motor</u>

The motor and inductance described in Appendices A and C respectively were used for the laboratory starting tests. The circuit used was that shown in Figure 2. The following comments pertain to the components used in the circuit:

a. A permanent magnet direct current generator was used for obtaining the instantaneous speed of the motor.

b. The inductance discharge resistor was set at about 23 ohms for

all runs.

c. An additional variable resistance was used in series with the inductance in order to vary the total external resistance of the circuit. The sum of this additional variable resistance and the resistance of the inductance was designated R. This additional resistance was measured at each setting by the resistance measuring circuit shown, using the ammeter-voltmeter method.

The actual procedure used for the laboratory runs was as follows:

a. Connect the circuit as shown in Figure 2.

b. Energize the shunt field, raise the current to a maximum value (about 1.8 amperes) and then reduce to 1.5 amperes which is the value used throughout the thesis.

c. Energize and adjust the Brush Recorder and the associated amplifiers.

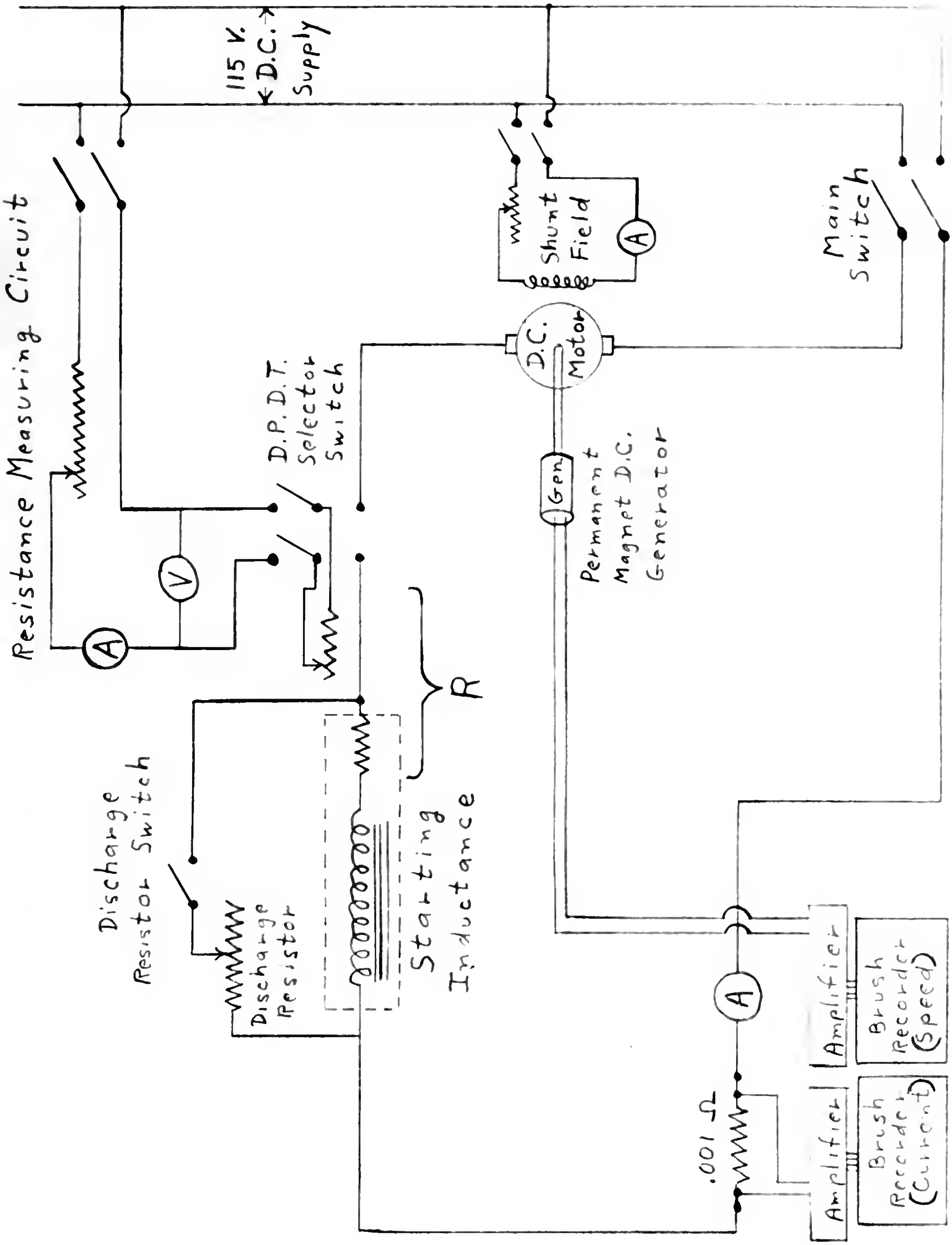
d. Set the resistance (R) to lowest value (1.8 ohms).

e. Open the discharge resistor switch.

f. Close the main switch to start the motor and observe the maximum current, steady-state current, and steady-state speed while recording instantaneous current and speed on the Brush Recorder.

g. After steady-state is reached, close discharge resistance switch and open the main switch.

h. Repeat steps d. through g. while increasing the induction



Experimental Starting Circuit
Figure 2

resistance (R) in steps of 0.2 ohm until critical damping was reached.

See Figures 3 through 8 for the instantaneous current and speed curves.

These runs were made with no external load on the motor. Thus, the load consisted of friction, windage and core loss.

4. Analog Computer Solution of Starting Circuit of Section 3

The motor constants were determined as outlined in Appendix A. It was found that the steady-state no load current of this motor was 5 amperes. This load was assumed to be a constant torque load and the torque was computed to be 3.73 lbs-ft. The initial armature current required to overcome static friction was 17 amperes.

The value of resistance and inductance for the starting inductance was found as described in Appendix C. A value of inductance (L) of 1.6 henrys was selected from Figure 25 as best representing the average value of inductance during the run excluding the initial current surge. The resistance used was 1.8 ohms.

The values of these parameters were then used to obtain the coefficients in the motor starting equations. These coefficients were utilized to obtain a solution for the instantaneous current and speed using the analog computer. The details of the procedure for the computer solution is contained in Appendix B.

Computer solutions were run for those conditions corresponding to the laboratory runs of Section 3. That is, the value of resistance (R)

was varied by increasing it in steps of 0.2 ohms from 1.8 ohms until critical damping was reached. These results are shown in Figures 3 through 8.

By comparing the computer solutions and the laboratory runs for the same value for resistance (R), it is seen that they are almost identical except for the initial current surge.

It is believed that the primary reason for the discrepency between actual and theoretical curves is the non-linearity of the inductance used in the laboratory.

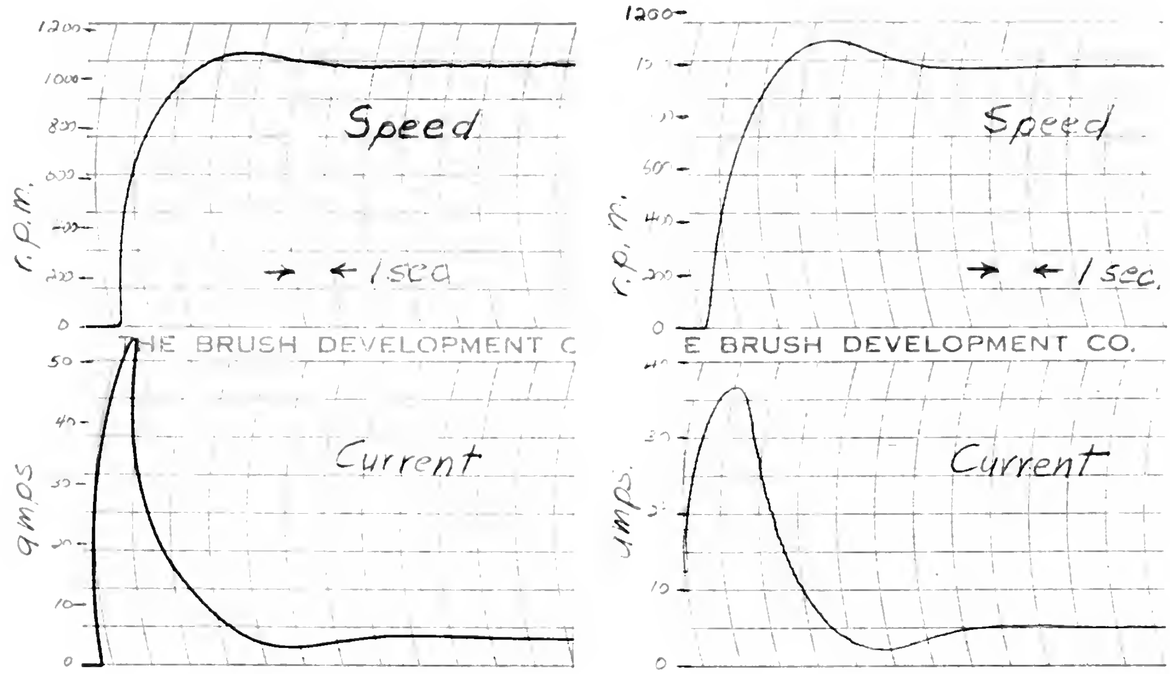
Comparing the experimental and theoretical curves, it is noted that, after the initial current surge:

a. All over and undershoots are of the same order.

b. The oscillation frequencies are almost identical.

c. Critical damping is reached at the same value of resistance (R) for both the theoretical and experimental case.

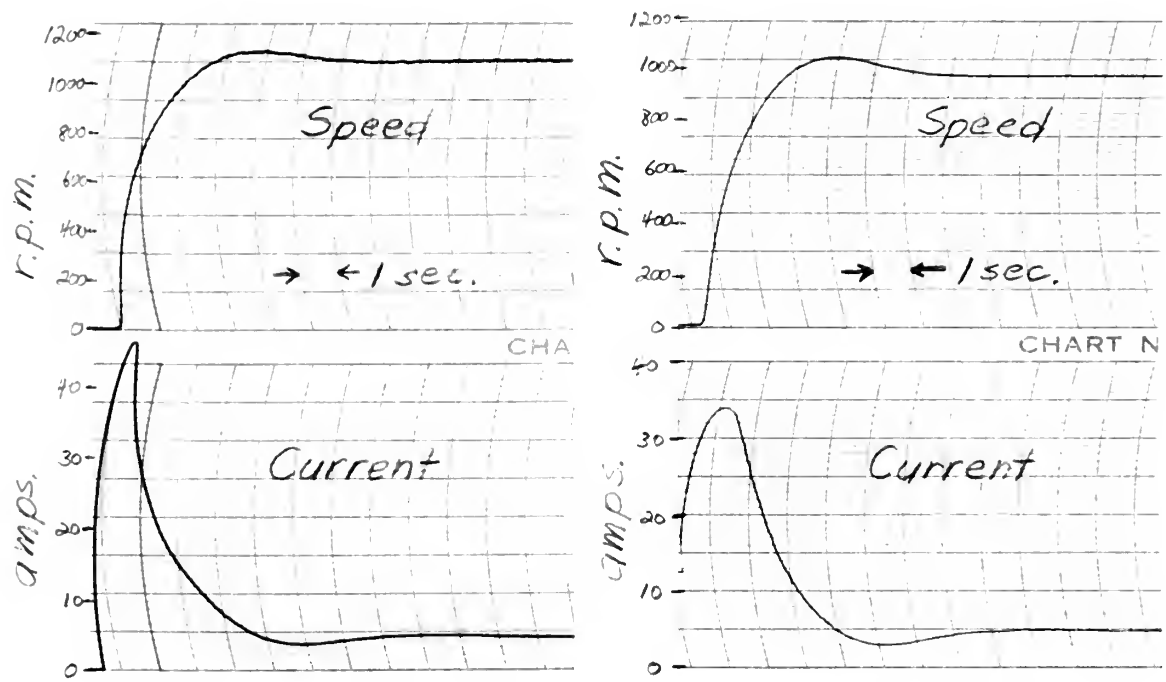
These results verify the theoretical equations and all constants used. Therefore, it is believed that an extension of the investigation of the effect of parameter variation on motor starting by analog computer is fully justified.



(a) Motor (b) Computer

Verification Runs, $R_t = 2.0$ ohms

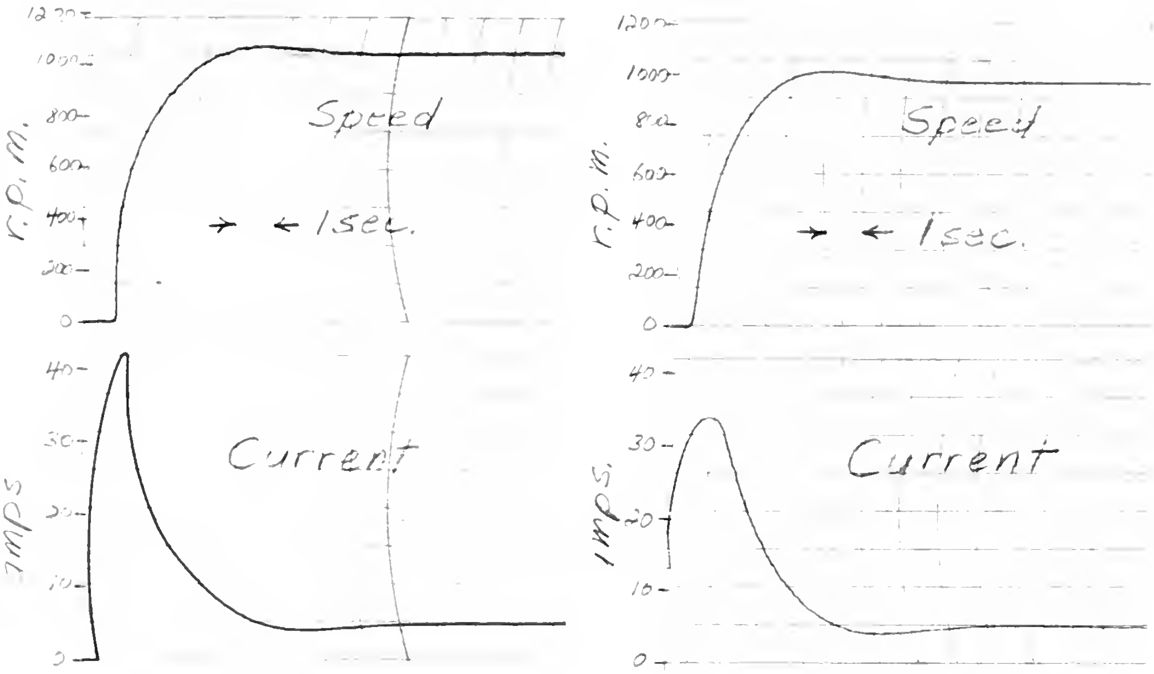
Figure 3



(a) Motor (b) Computer

Verification Runs, $R_t = 2.2$ ohms

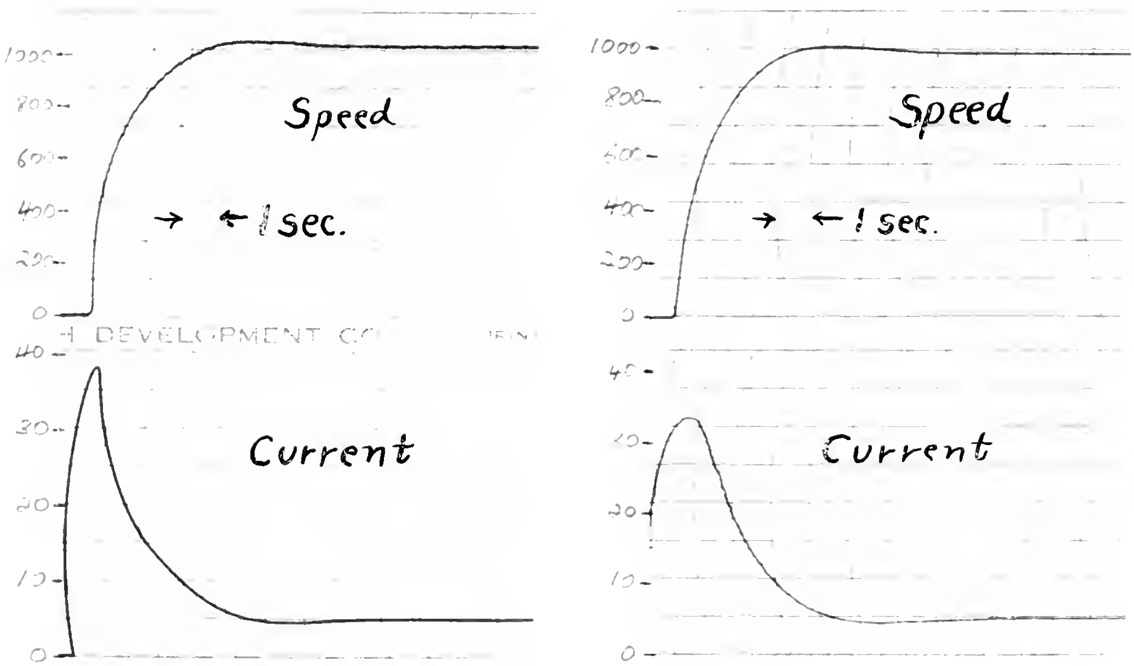
Figure 4



(a) Motor (b) Computer

Verification Runs, R_t 2.4 ohms

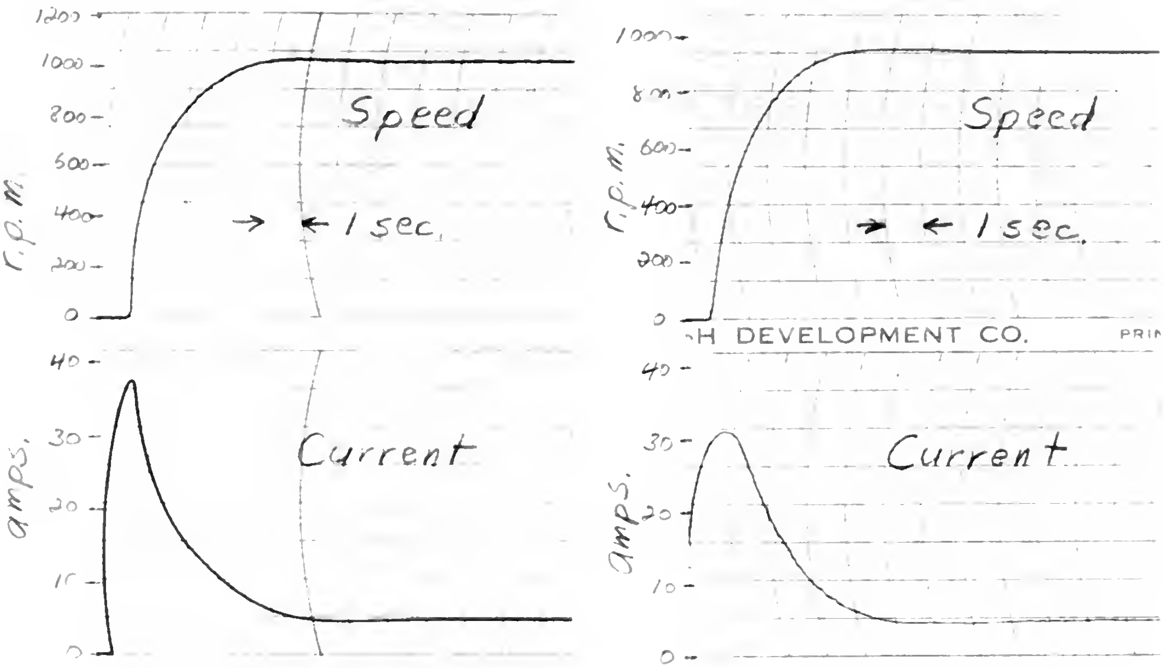
Figure 5



(a) Motor (b) Computer

Verification Runs, R_t = 2.6 ohms

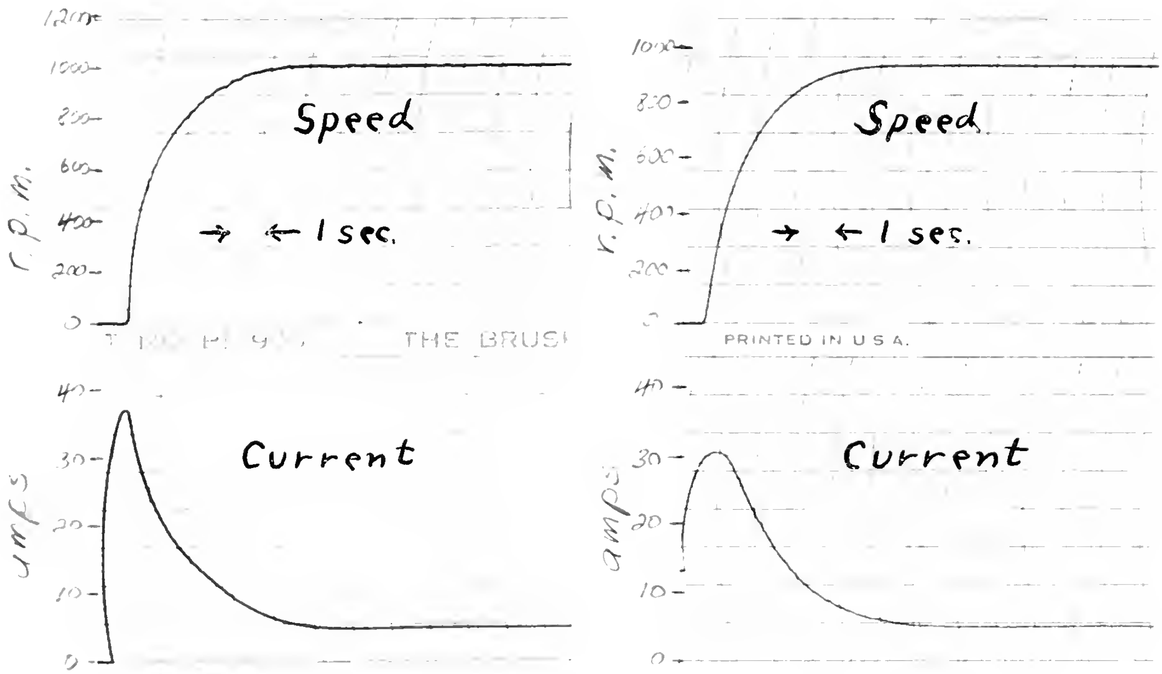
Figure 6



(a) Motor (b) Computer

Verification Runs, $R_t = 2.8$ ohms

Figure 7



(a) Motor (b) Computer

Verification Runs, $R_t = 3.0$ ohms

Figure 8

5. Effect of Variation of Parameters on Inductive Starting of Direct Current Motors

The details for obtaining the analog computer solutions are covered in Section 4 above and in Appendix B. In this section it is desired to summarize the runs made and show graphically the effect of the variation of parameters. The solutions for current and speed for the various runs are included in Appendix D.

In order to show graphically the effect of varying the parameters, the following data, where applicable, were obtained from the solutions in Appendix D:

a. Peak overshoots of current and speed. (% of steady-state values)

b. Period of oscillations of current and speed.

c. Log decrement of current and speed oscillations. (Log decrement is defined as the natural logarithm of the ratio of the excursions during two successive oscillations)

Figures 9 and 10 are a tabulation of computer calculations made and give the figure numbers for the resulting curves.

Summary of Computer Solutions

Constant Torque Load

Load Torque (Tc - lbs-ft)	Inertia (J-slug-ft^2)	Inductance (L-henrys)	Figure No.	Remarks
44.76	0.409	1	26	Performance summarized in Fig. 11 and Fig. 12
44.76	0.409	2	27	
44.76	0.409	3	28	
44.76	0.409	4	29	
44.76	0.409	5	30	
44.76	0.300	1	31	Performance summarized in Fig. 13 and Fig. 14
44.76	0.409	1	26	
44.76	0.500	1	32	
44.76	0.600	1	33	
44.76	0.700	1	34	
44.76	0.818	1	35	
44.76	0.204	2	36	
44.76	0.409	2	27	
44.76	0.818	2	37	
44.76	0.204	3	38	
44.76	0.409	3	28	
44.76	0.818	3	39	
11.19	0.409	2	40	Performance summarized in Fig. 15 and Fig. 16
22.38	0.409	2	41	
33.57	0.409	2	42	
44.76	0.409	2	27	
11.19	0.409	3	43	
22.38	0.409	3	44	
33.57	0.409	3	45	
44.76	0.409	3	28	
11.19	0.409	4	46	
22.38	0.409	4	47	
33.57	0.409	4	48	
44.76	0.409	4	29	
11.19	0.409	5	49	

Figure 9

Summary of Computer Solutions (Cont.)

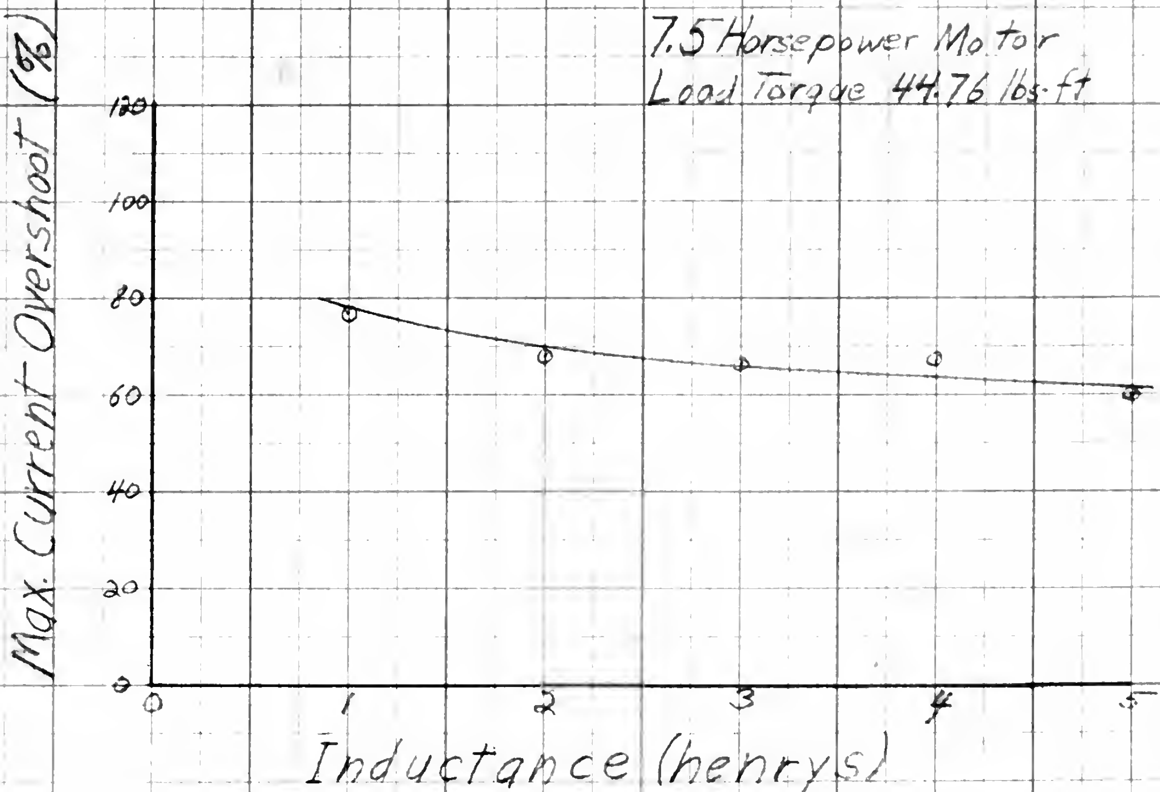
Constant Torque Load (Cont.)

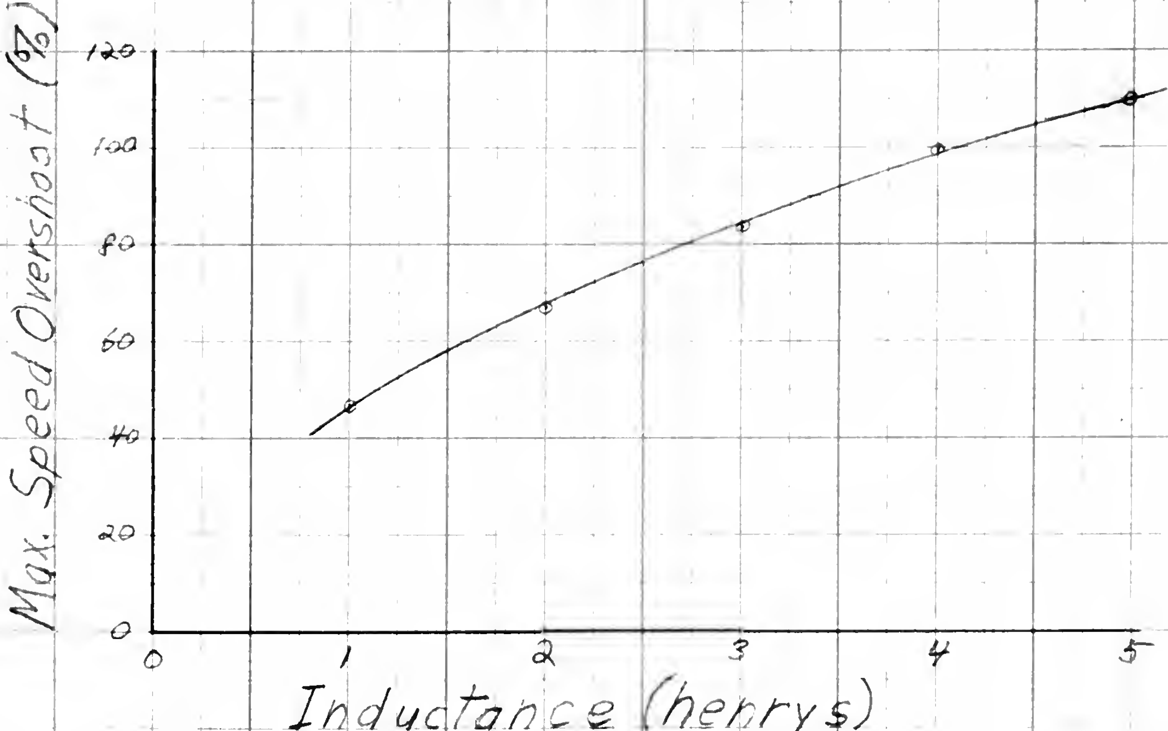
Load Torque (T_c - lbs-ft)	Inertia (J - slug-ft^2)	Inductance (L - henrys)	Figure No.	Remarks
22.38	0.409	5	50	Performance summarized in Figures 15 and 16
33.57	0.409	5	51	
44.76	0.409	5	30	

Torque Load Proportional to Speed

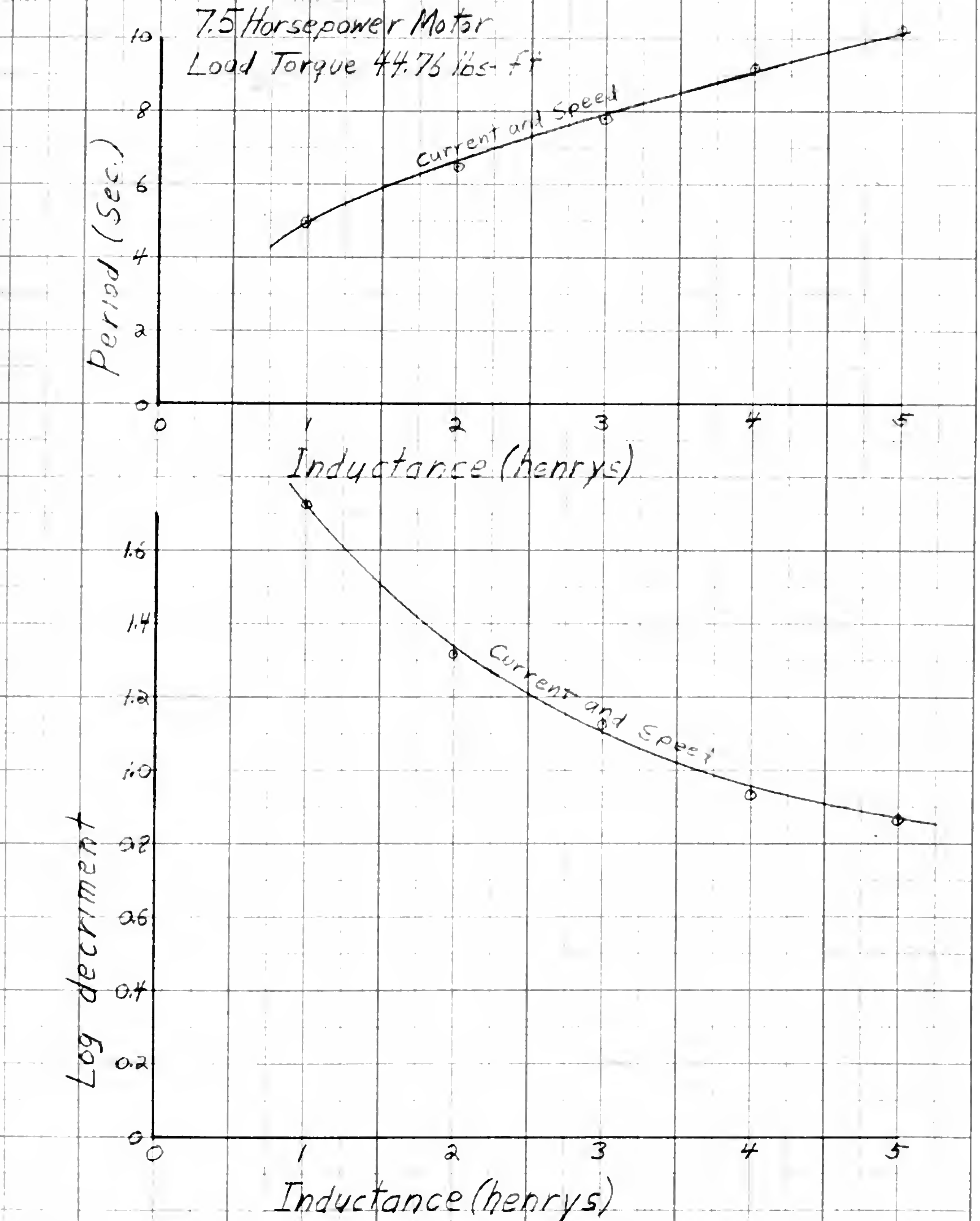
Load Torque (T_s - lbs-ft/rad/sec)	Inertia (J - slug-ft^2)	Inductance (L - henrys)	Figure No.	Remarks
0.621	0.409	1	52	Performance summarized in Figure 17
0.621	0.409	2	53	
0.621	0.409	3	54	
0.155	0.409	1	55	Performance summarized in Figure 18
0.310	0.409	1	56	
0.466	0.409	1	57	
0.621	0.409	1	52	
0.155	0.409	2	58	
0.310	0.409	2	59	
0.466	0.409	2	60	
0.621	0.409	2	53	
0.155	0.409	3	61	
0.310	0.409	3	62	
0.466	0.409	3	63	
0.621	0.409	3	54	

Figure 10



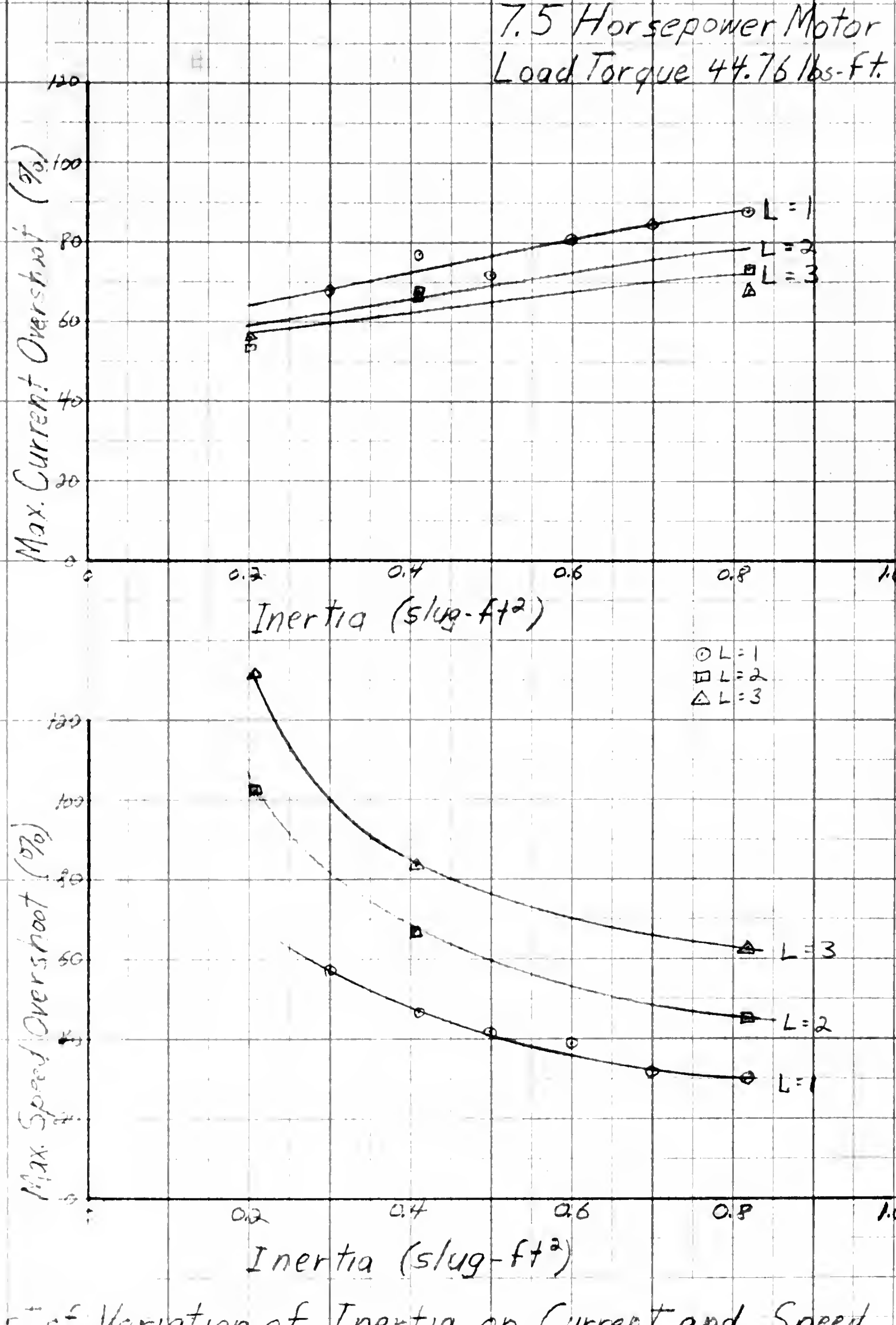


Effect of Variation of Inductance on Current and Speed Overshoot, Constant Torque Load
Figure 11



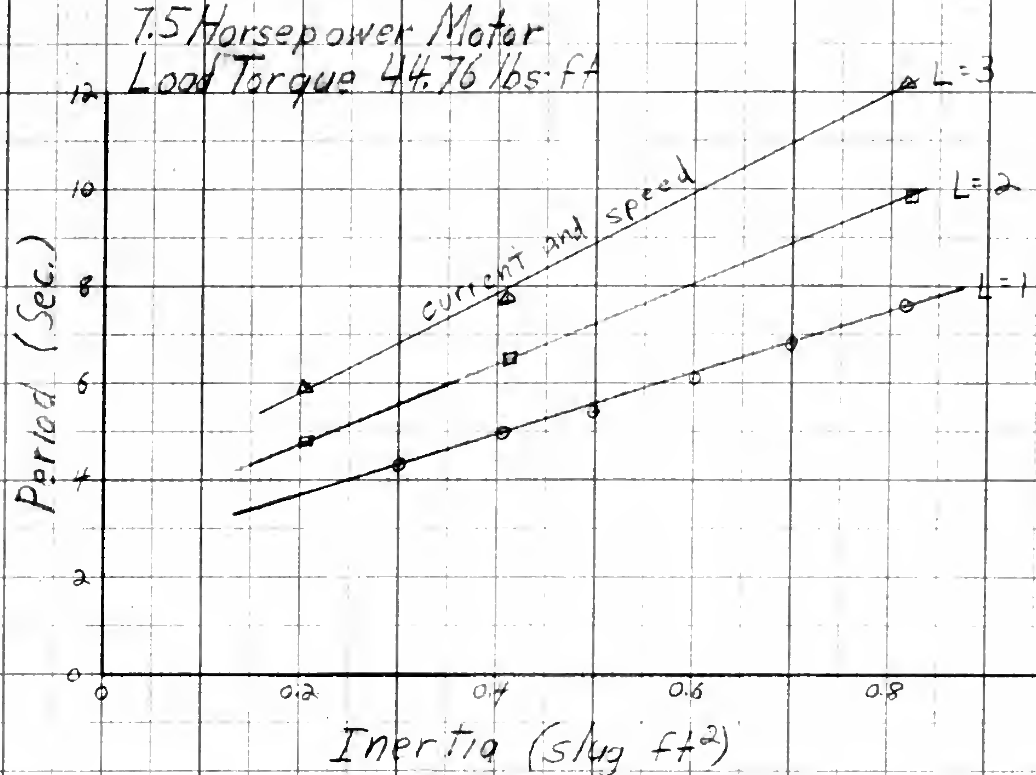
Effect of Variation of Inductance on Current and Speed Oscillations, Constant Torque Load

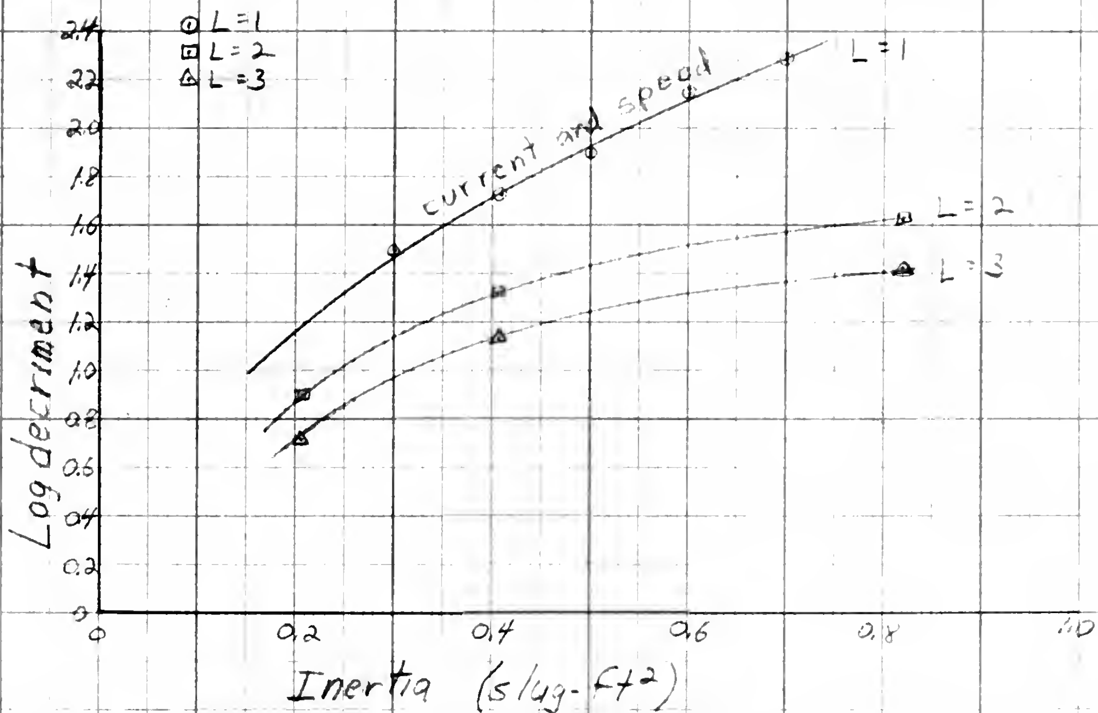
Figure 12



Effect of Variation of Inertia on Current and Speed Overshoot, Constant Torque Load

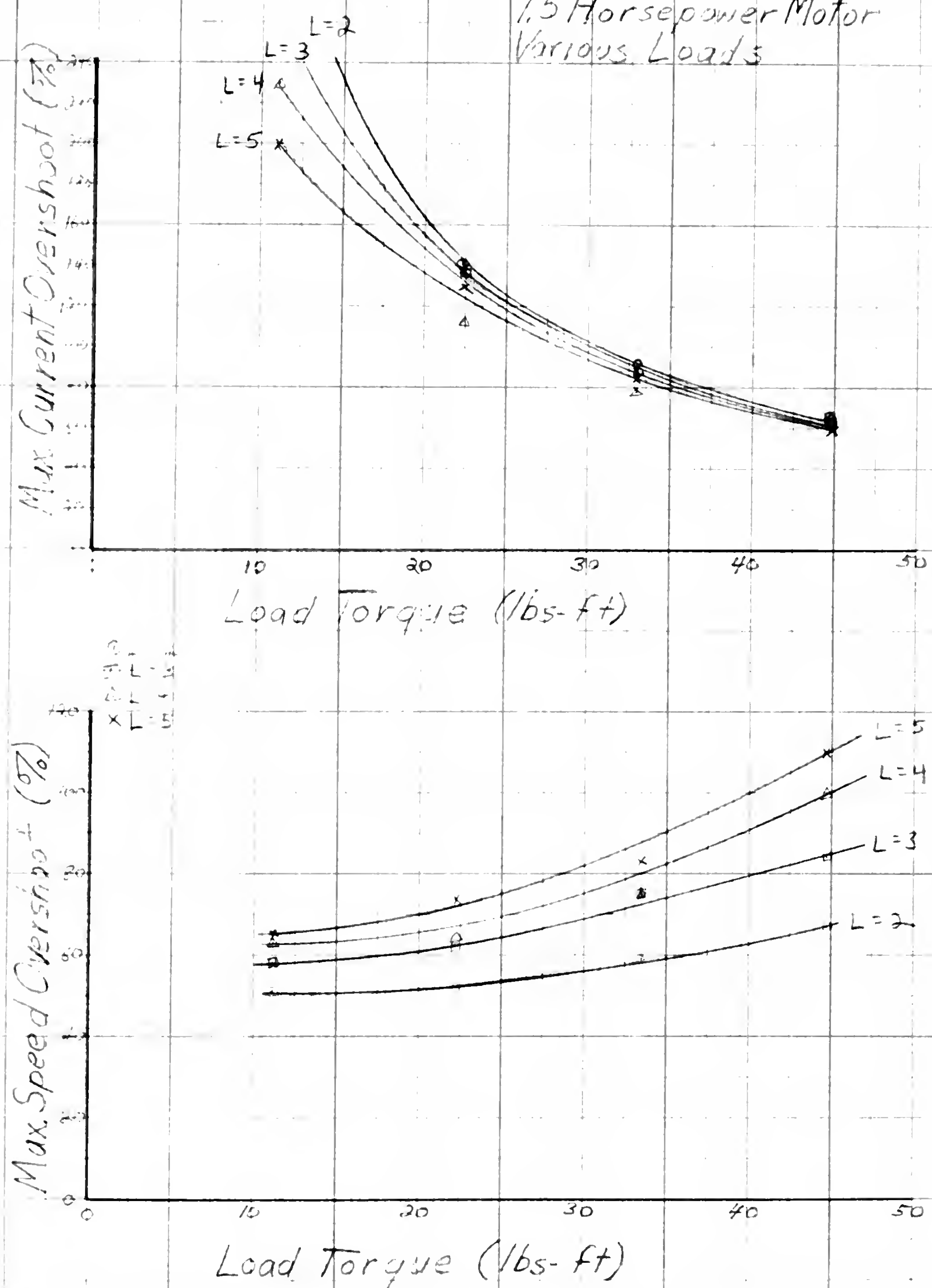
Figure 13





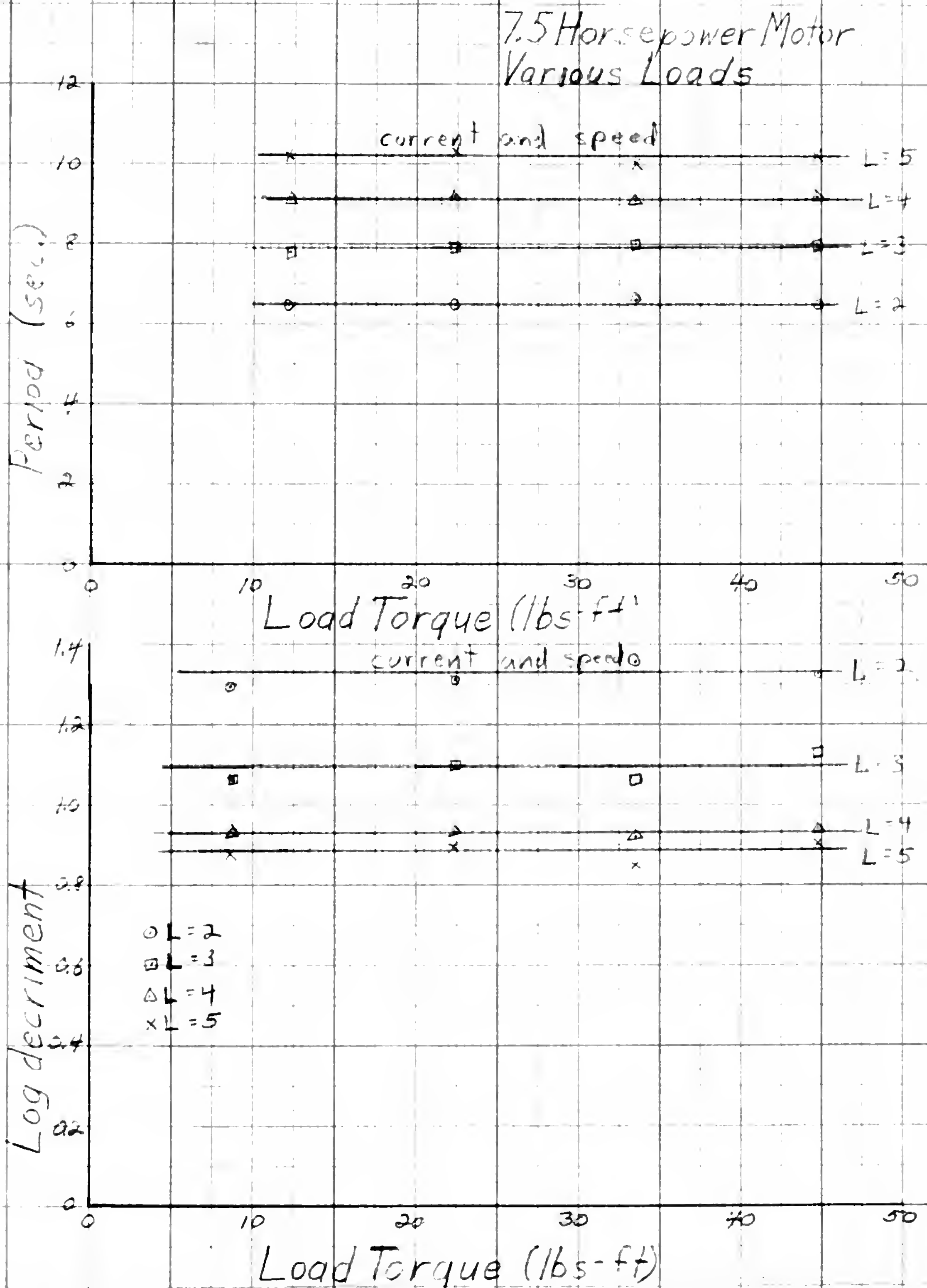
Effect of Variation of Inertia on Current and Speed Oscillators, Constant Torque Load.

Figure 14



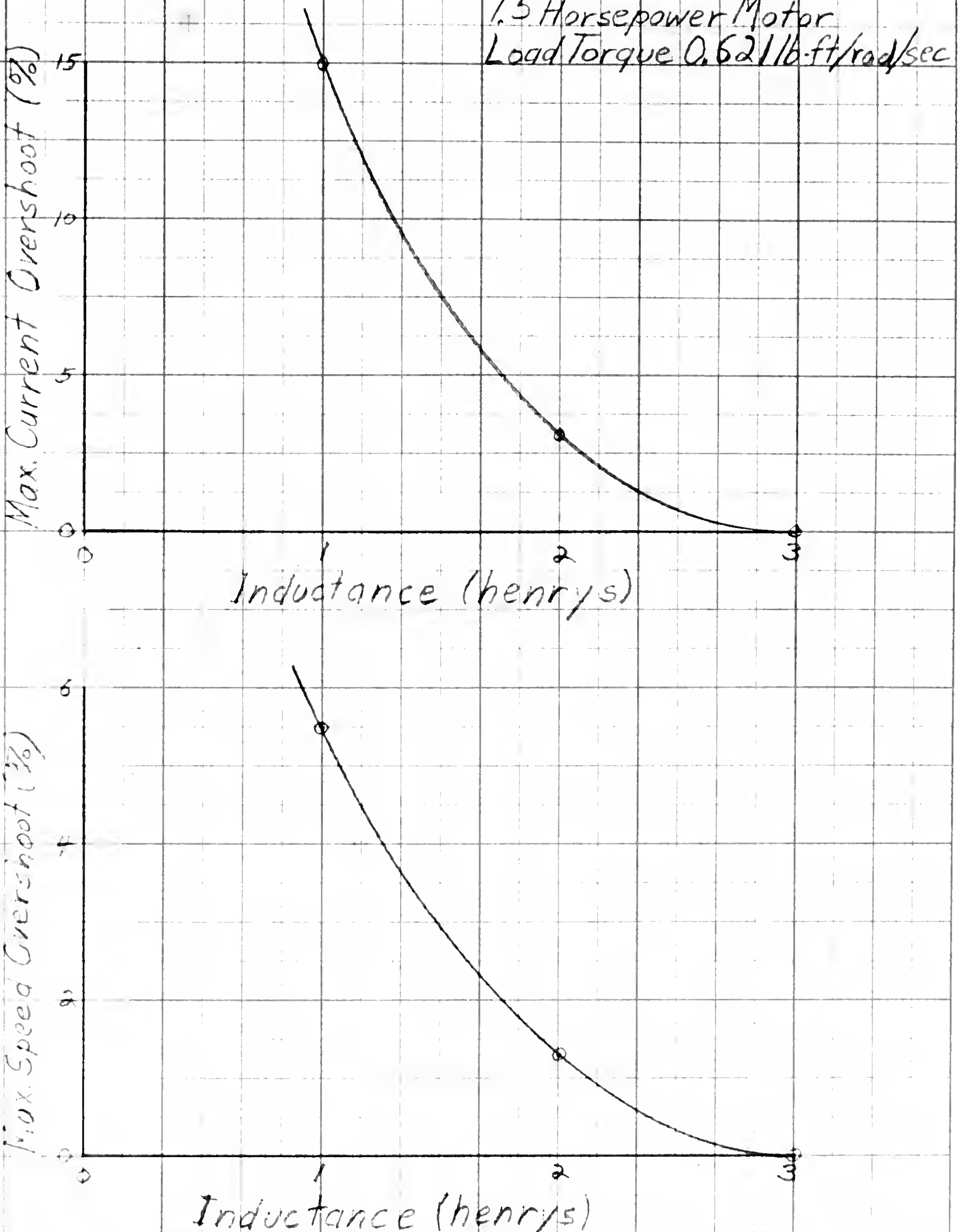
Effect of Variation of Load on Current and Speed Overshoot, Constant Torque Load

Figure 15



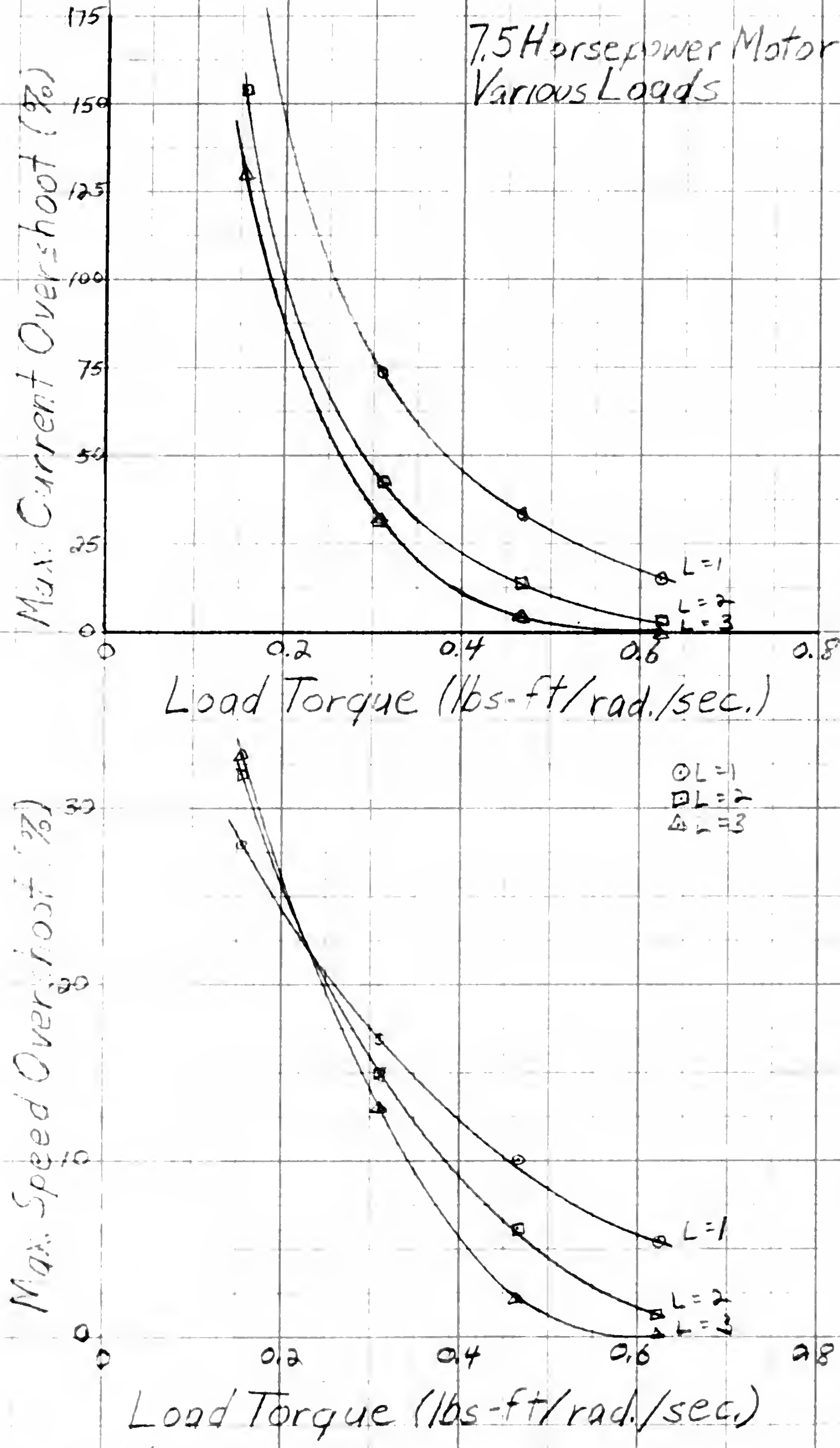
Effect of Variation of Load on Current and Speed Oscillations, Constant Torque Load

Figure 16



Effect of Variation of Inductance on Current and Speed Overshoot, Load Torque proportional to Speed

Figure 17



Effect of Variation of Load on Current and Speed Overshoot, Load Torque proportional to Speed

Figure 18

CHAPTER IV

CONCLUSIONS

This investigation was limited to the inductive starting of a 7.5 horsepower, 110 volt, direct current, shunt motor. The following conclusions are based on the results of this investigation:

a. An inductance of one henry with a resistance of 0.5 ohms will limit the starting current to an acceptable value. (Current overshoot less than 100% of steady-state value)

b. Some current and speed oscillations will occur, but the magnitudes and periods are considered acceptable. (Speed overshoot less than 50% of steady-state value, period 5 sec., log decrement 1.7)

c. The period of the oscillations can be lengthened without detrimental effects by increasing the inertia of the rotating parts.

d. Increasing the value of inductance results in an increase of magnitude and persistance of the speed oscillation.

e. The type of motor load will materially affect the starting characteristics. The worst oscillations occur when the load torque remains constant, independent of speed. A load torque which is proportional to the angular velocity has a damping effect, the greater the load the greater the damping.

Changing the resistance of the armature circuit could lead to markedly different results, and further investigation along this line

should be conducted.

It is believed that the size and weight of an inductive starter would limit its application. It appears to be most desirable for use with a motor that is subjected to repeated starting and stopping, and least desirable where space and weight are critical. An investigation of the optimum design of a starting inductance is necessary to provide more information on size, weight and cost.

No attempt has been made to predict the performance of motors with different power ratings since the armature resistance varies considerably and it is an important parameter in the determination of starting characteristics. It is recommended that such an investigation be conducted. The analog computer proved to be a very convenient device for performing this type of investigation. The results from the computer are not absolutely correct, but the speed and ease of obtaining solutions for a wide range of parameters far outweigh the slight inaccuracies in the solution.

BIBLIOGRAPHY

1. Minner, Donald A., and Parker B. Armstrong. Inductive Starting of Direct Current Motors, U. S. Naval Postgraduate School, Annapolis, Md., 1948.

APPENDIX A

DETERMINATION OF MOTOR CONSTANTS

The motor used was a Westinghouse Type SK D.C. Marine Motor, shunt wound, style 9E218, serial 4702997, 7.5 HP, 850/1200 rpm, 115 volt, 59 amperes. The motor was coupled to a Westinghouse 5 KVA Generator, S.O. 9E217, serial 4703012, 60 cycle, 110/220 volts, 1200 rpm. The following values were used for motor constants:

K_g - - - - - - - - - - - 1.012 volts/radian/second

K_t - - - - - - - - - - - 0.746 lbs-ft/amp

J - - - - - - - - - - - 0.409 slug-ft^2 (Includes the rotor of the 5 KVA generator)

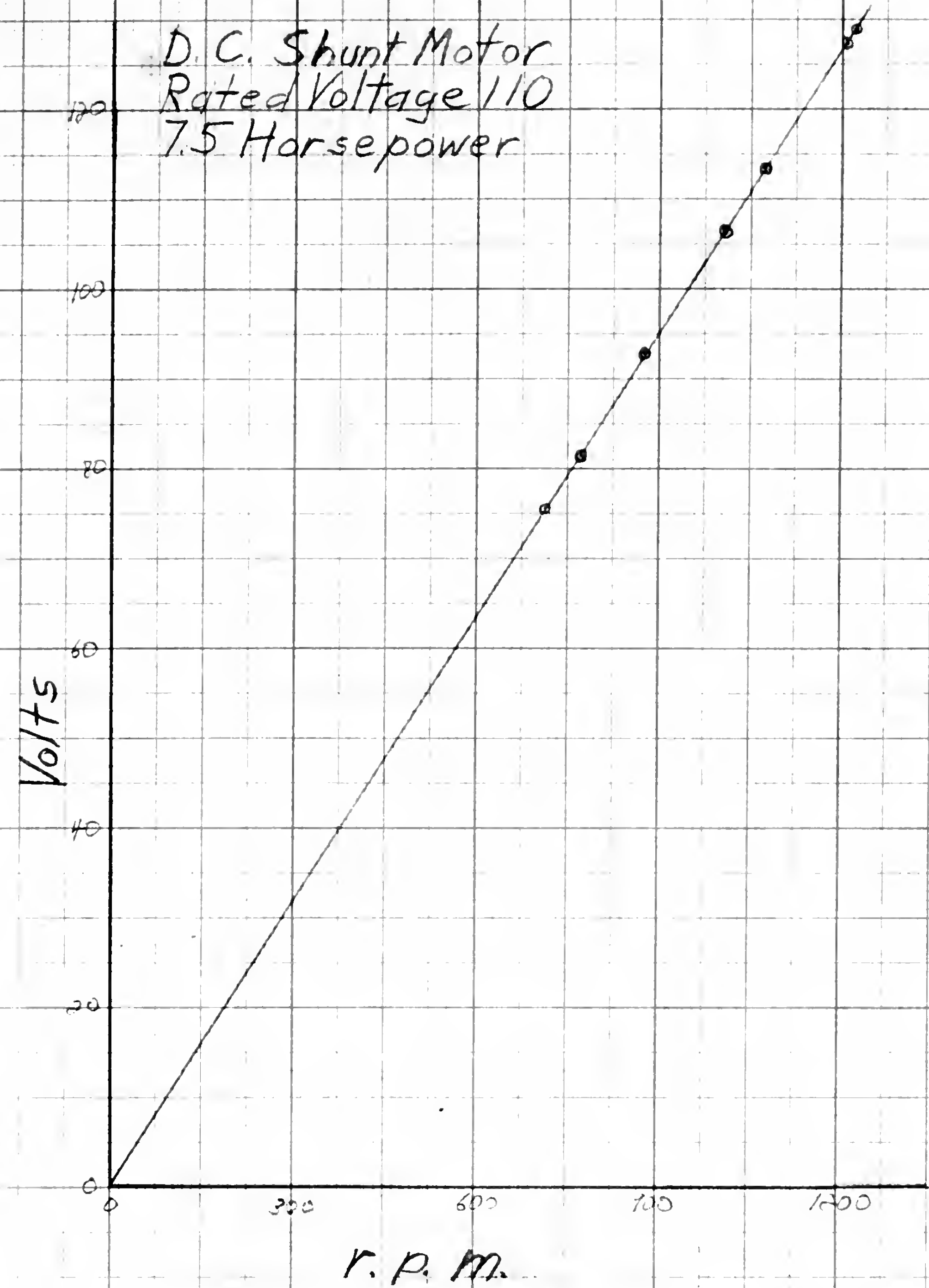
R_a - - - - - - - - - - - 0.2 ohms

L_a - - - - - - - - - - - 0.0 henrys

For all motor tests and laboratory runs the field was separately excited. The field current was increased to its maximum value, reduced to 1.5 amperes, and then held constant.

The motor back e.m.f. constant (K_g) was determined by driving the motor as a generator and measuring the open circuit voltage at various speeds. A curve of the results is shown in Figure 19.

The motor torque constant (K_t) was computed as follows: The developed power at steady state is equal to the product of developed torque and motor speed, and is also equal to the power input to the armature less copper loss.



Generated Back E.M.F. vs Motor Speed
for Constant Field Current (1.5 amps.)

Figure 19

$$T\omega = (0.7376)(IE - I^2 R_t), \tag{1}$$

where 0.7376 is the conversion factor from watts to lbs-ft per second.

$$T = K_t I \qquad \text{(assuming field flux remains constant)} \tag{2}$$

from equation (4) of Chapter II. Substituting equation (2) in (1) for T and dividing by I,

$$K_t \omega = 0.7376(E - IR_t) \tag{3}$$

At steady state,

$$E - I R_t = K_g \omega \tag{4}$$

from equation (3) of Chapter II. Substituting equation (4) in (3) for $E - IR_t$ and dividing by ω,

$$K_t = 0.7376 K_g. \tag{5}$$

The moment of inertia was measured by means of a retardation test. With the motor running at no load under steady state conditions, the armature circuit was opened and the transient speed versus time recorded. Under these conditions

$$J \frac{d\omega}{dt} + F(\omega) = 0 \tag{6}$$

where $F(\omega)$ represents the torque required to overcome friction, windage, and core loss.

$F(\omega)$ was determined by running the motor at no load with a constant, separately excited field, varying the terminal voltage, and recording the steady state armature current and motor speed. A curve plotted from these data is shown in Figure 20. For speeds over 500 rpm, the current remained constant so

$$F(\omega) = K_t I = 3.73 \text{ ft lbs.} \tag{7}$$

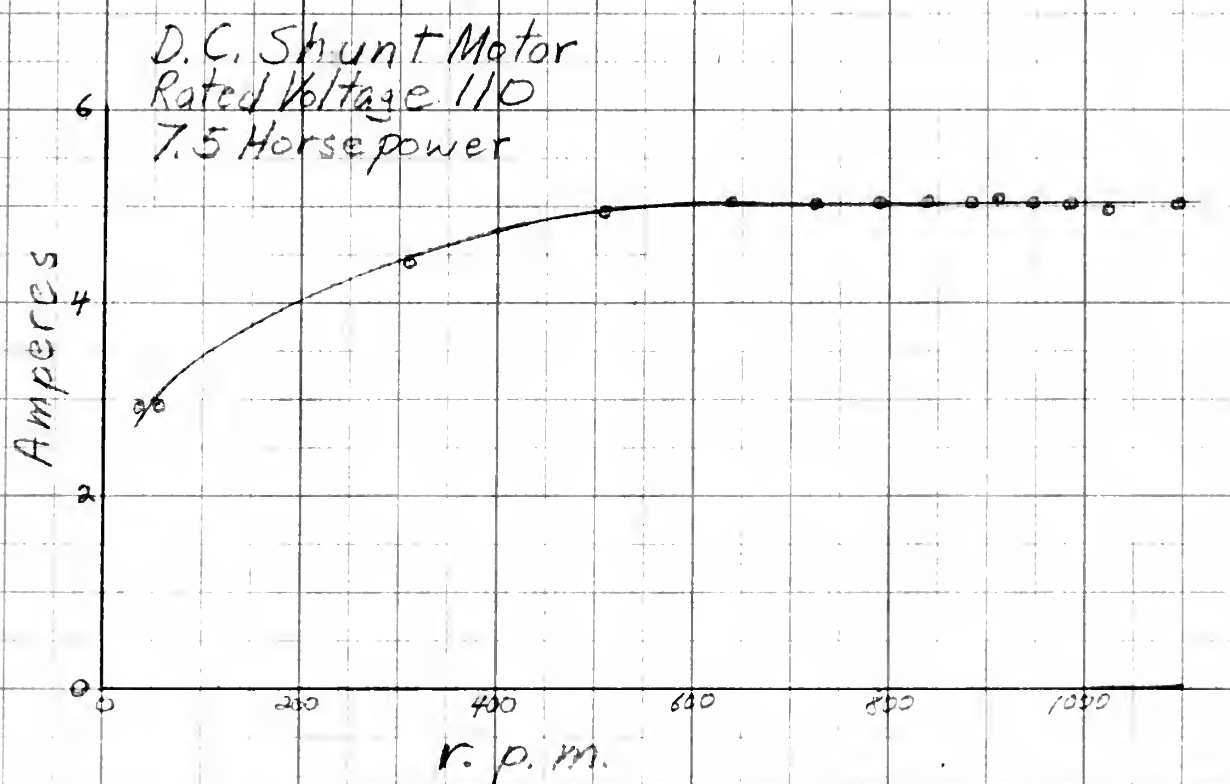
Several retardation curves were made and the slopes measured between 1100 and 500 rpm. The average value of $\frac{d\omega}{dt}$ was -9.12 radians/sec./sec.

$$J = -\frac{F(\omega)}{d\omega/dt} \tag{8}$$

$$J = .409 \tag{9}$$

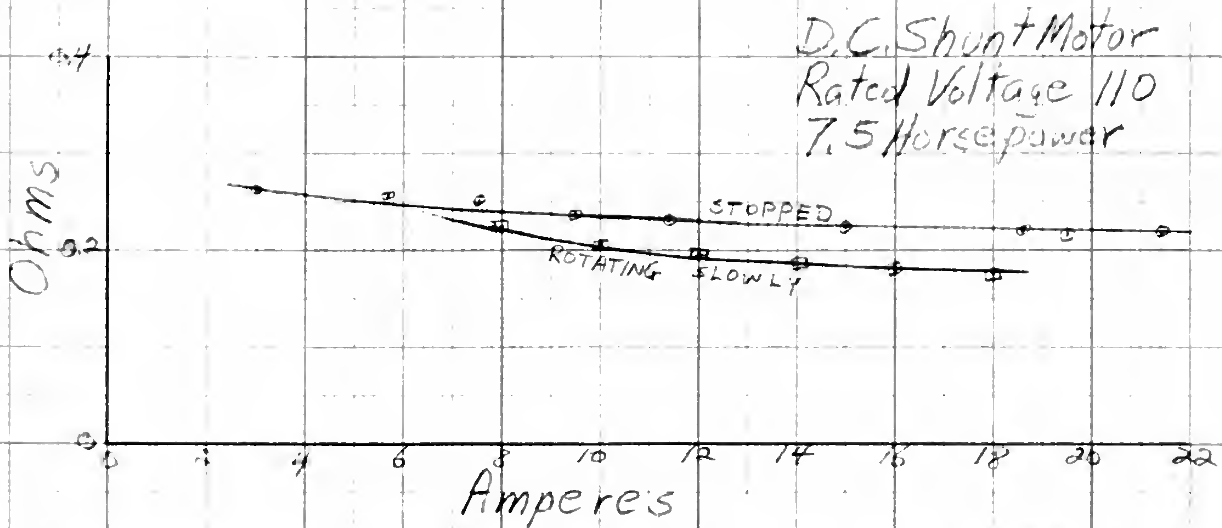
The armature resistance was measured at various currents by the voltmeter-ammeter method, with the armature stopped and also rotating slowly. Curves of the results are shown in Figure 21. A value of 0.2 ohms was selected for use in computations.

The armature inductance is small and it was considered to be negligible compared to the values of external inductance used.



Steady-state Armature Current vs Motor Speed at No Load with Constant Field Current (1.5 amps)

Figure 20



Effective Armature Resistance vs Armature Current

Figure 21

APPENDIX B

SOLUTION OF MOTOR STARTING EQUATIONS BY ANALOG COMPUTER

A Boeing Electronic Analog Computer, Model 6631, serial no. 5127, manufactured by the Boeing Airplane Company of Seattle, Washington, was used for the solution of the motor starting equations.

The following equations were solved:

$$\frac{d^2\omega}{dt^2} + \left(\frac{T_s}{J} + \frac{R_t}{L_t}\right)\frac{d\omega}{dt} + \left(\frac{R_t T_s}{L_t J} + \frac{K_g K_t}{L_t J}\right)\omega + \frac{R_t T_c}{L_t J} - \frac{E K_t}{L_t J} = 0 \qquad (1)$$

The initial conditions for this equation are:

$$\omega = 0\,, \quad \frac{d\omega}{dt} = \frac{i_o K_t - T_c}{J}$$

$$i = \frac{J}{K_t}\frac{d\omega}{dt} + \frac{T_s}{K_t}\omega + \frac{T_c}{K_t} \qquad (2)$$

The development of these equations is given in Chapter II. It is assumed that all circuit components are linear.

In order to simplify the notation for the following discussion, the coefficients in equations (1) and (2) above are designated as:

$$A = \frac{T_s}{J} + \frac{R_t}{L_t}$$

$$B = \frac{R_t T_s}{L_t J} + \frac{K_g K_t}{L_t J}$$

$$C = \frac{R_t T_c}{L_t J} - \frac{E K_t}{L_t J}$$

$$D = J/K_t$$

$$E = T_s/K_t$$

$$F = T_c/K_t$$

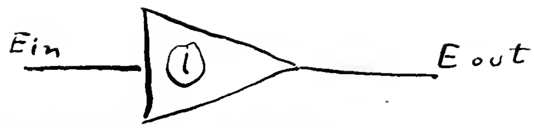
The equations then become:

$$\frac{d^2 w}{dt^2} + A\frac{dw}{dt} + Bw + C = 0 \qquad (3)$$

$$i = D\frac{dw}{dt} + Ew + F \qquad (4)$$

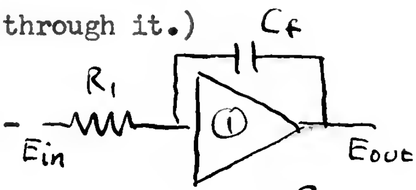
The following circuit symbols are used in the schematic diagram of the analog computer:

Amplifier -



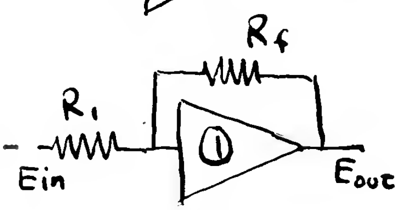
(The amplifier changes the sign of any item that is sent through it.)

For Integration -



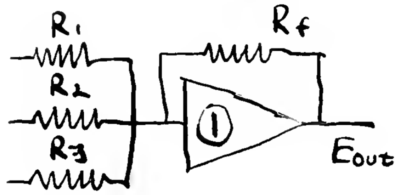
$$E_{out} = -\frac{1}{C_f}\int \frac{E_{in}}{R_1}\,dt$$

For Multiplication -



$$E_{out} = -\frac{R_f}{R_1}E_{in}$$

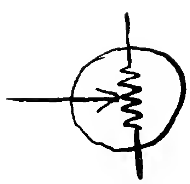
For Addition -



$$E_{out} = -R_f\left[\frac{E_{in_1}}{R_1} + \frac{E_{in_2}}{R_2} + \frac{E_{in_3}}{R_3}\right]$$

(This is simply a multiplication device with more than one input.)

Coefficient Potentiometer -



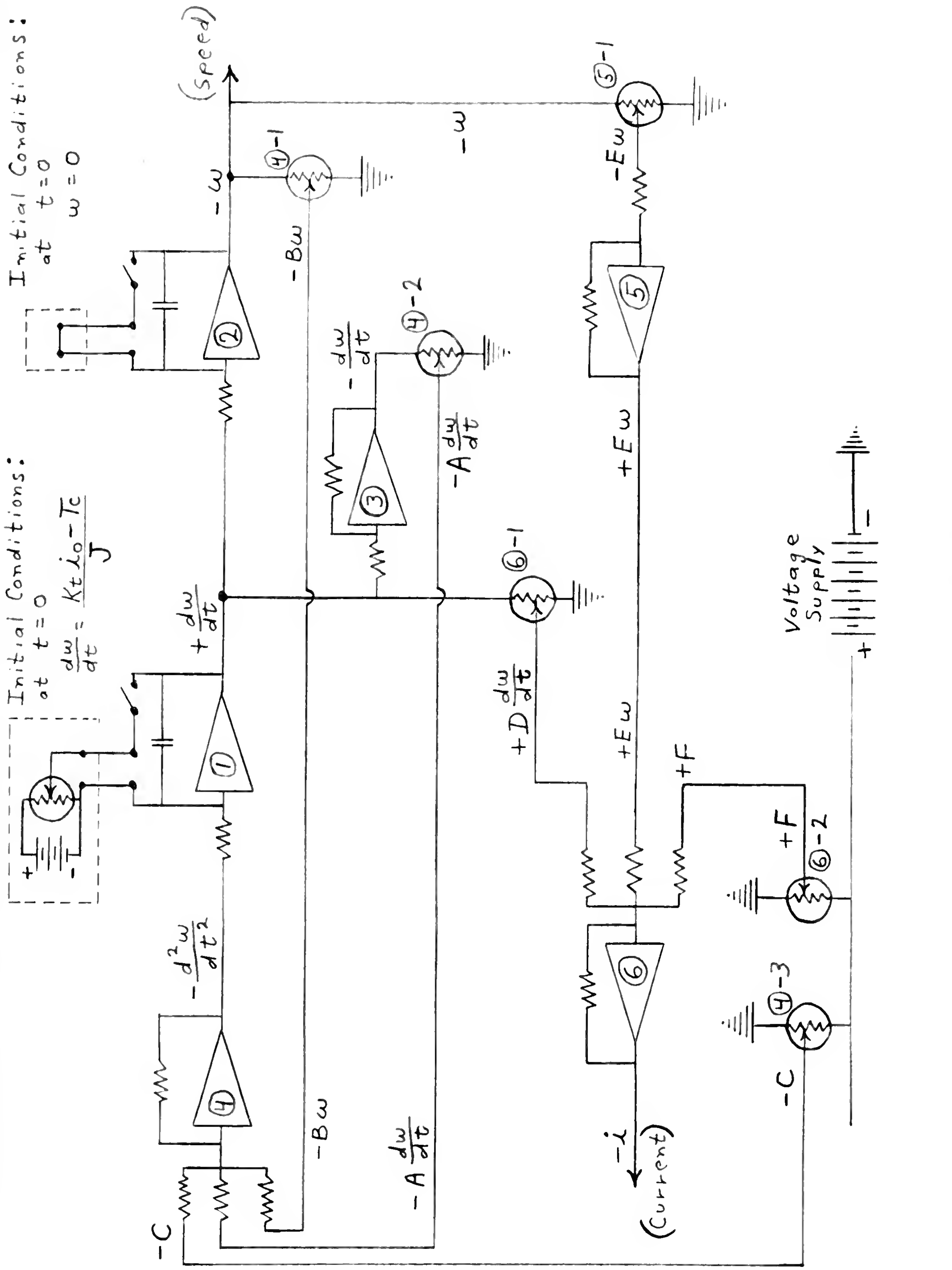
1 - 1

Equation (3) was rearranged as follows in order to readily set it up on the computer:

$$A\frac{d\omega}{dt} + B\omega + C = -\frac{d^2\omega}{dt^2} \tag{5}$$

The current equation was used in the form shown in equation (4) above. The computer schematic diagram for the solution of these equations is shown in Figure 22. The initial conditions were imposed on amplifiers (1) and (2) as shown. The instantaneous values of current (i) and speed (ω) were the outputs of amplifiers (6) and (2) respectively. These outputs were used to drive Brush Recorders and gave graphical solutions which appear in Chapter III and Appendix D.

In order to obtain a particular solution, the values of all parameters were used to compute the various coefficients A through F. The input resistors of all amplifiers were one megohm. The feedback condensers of amplifiers (1) and (2) were one microfarad, and the feedback resistors of amplifiers (3) and (5) were one megohm. The values of the feedback resistors of amplifiers (4) and (6) and the settings of the potentiometers were then determined from the values of these coefficients.



Schematic Circuit Diagram for the Analog Computer
Figure 22

APPENDIX C

INDUCTANCE USED FOR STARTING THE DIRECT CURRENT MOTOR

In order to verify the analog computer solutions of the motor starting equations, an inductance was designed and built for use in the laboratory to start the 7.5 horsepower motor described in Appendix A. Initially it was hoped that an inductance could be found with the following general specifications:

The value of the inductance to be at least one henry.

The value of the resistance to be 0.5 ohms or less.

The value of the inductance to remain constant over a range of currents to at least 60 amps.

The inductance obtained differed considerably from these original specifications.

It is readily seen from basic inductance theory, that if an air-core inductance were to be used, a great many turns would be required. And in order to fulfill the resistance requirement, the cross section of the conductor would have to be large. This would increase the winding problem, increase the mean length of each turn and cause the physical size of the inductance to be prohibitive.

If a closed iron core inductance is used in order to reduce the number of turns required, the problem of saturation is immediately encountered. In order to maintain a reasonably constant inductance without marked saturation effects to 60 amps, the cross section of the

core would be quite large. This would necessitate a large core in order to accommodate the turns required. The cross-sectional area of the conductor would be large to give the desired resistance. The physical size of this type of inductance would be just as prohibitive as the air core inductance. The savings in the number of turns would be offset by the increased length of each turn due to the large area of the iron core. Thus, very little, if any, net advantage would be achieved.

The inductance built was a compromise between the above two extremes, and the specifications were relaxed considerably. An iron core was used in order to reduce the number of turns required. However, a large air gap was used to allow the current to reach about 30 amperes before saturation greatly reduced the value of the inductance. The value of resistance was kept as low as possible consistant with equipment and funds available.

At the suggestion of Professor O. H. Polk, it was decided to use a laminated core so that the inductance obtained could be retained in the Electrical Engineering Laboratory of the U. S. Naval Postgraduate School, and put to practical use on both alternating and direct current circuits.

The inductance finally used was built by modifying a single phase transformer. The transformer used was an obsolete 75 KVA General Electric Type H. It consisted of the following windings on a distributed type core:

a. The high voltage winding consisted of two sections. Each section contained 220 turns.

b. The low voltage winding consisted of two sections. Each section contained 22 turns, six conductors per turn connected in parallel.

These windings were modified as follows:

a. The outer section of the low voltage winding consisting of 22 turns was removed.

b. Five layers averaging 107 turns of no. 8 magnet wire per layer were wound in this space.

c. Five conductors of the remaining low voltage section were reconnected in series to give 121 turns.

d. The high voltage winding was left intact.

The winding used consisted of seven separate coils with their terminals brought out and connected in series. The total number of turns was 1093.

The core was modified by cutting through each leg. This cut was made in a position which allowed considerable adjustment of the width of the air gap. Figure 23 shows the inductance as it was used in the laboratory.

The next step was to determine the value of the inductance and resistance at various values of current. Two methods were used.

The first method was to test the inductance by imposing a step voltage across the terminals and recording the transient current on a Brush Recorder. For any particular value of current, the inductance was determined by using the equation:

$$L = \frac{E - iR}{di/dt} .$$

The steps used to measure the inductance were:

a. The coil resistance was measured by the voltmeter-ammeter method.

b. A step voltage was impressed on the circuit and the transient current was recorded.

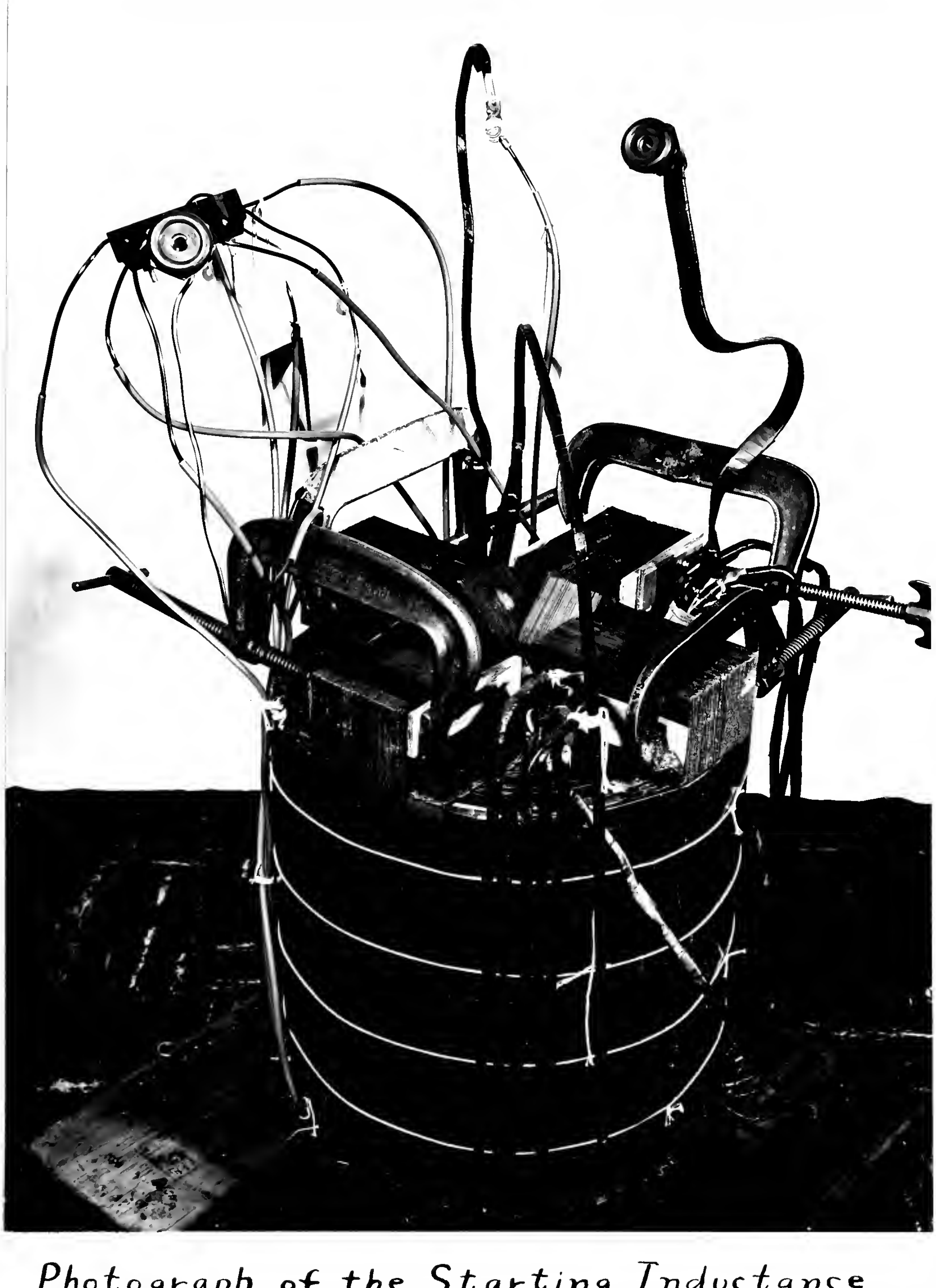
c. At each value of current (i), for which L was desired, the slope ($\frac{di}{dt}$) of the transient curve and the value of iR were determined.

d. Values of E, iR and $\frac{di}{dt}$ were substituted in the equation for the inductance and the equation was solved for L.

This method has a definite limitation in accuracy due to the difficulty in measuring the slope of the transient curve. However, this method was used to establish the method of connecting the seven coils of the inductance and of adjusting the air gap size to get an acceptable value of inductance with satisfactory saturation characteristics. The value of resistance (R) was 1.8 ohms. The values of inductance (L) obtained by this method are plotted in Figure 25.

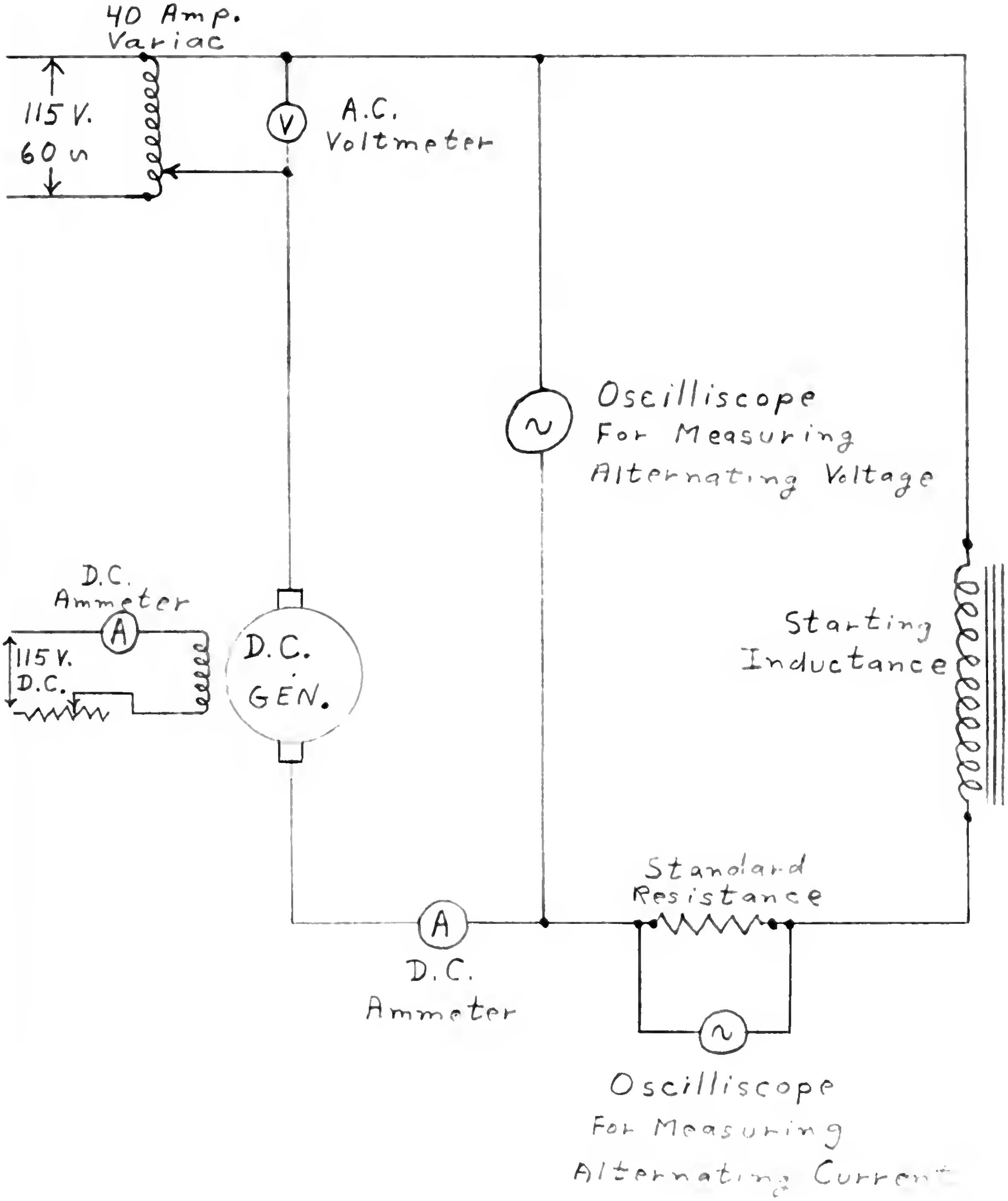
The second method was to place a D.C. voltage on the inductance and to adjust this voltage to obtain a direct current of the value at which it was desired to measure the inductance. Then an alternating voltage of about 50 volts was superimposed upon the D.C. voltage. The alternating current through the inductance, and the alternating voltage drop across the inductance were measured. From these two alternating values, the impedance was computed. Knowing the frequency of the

alternating voltage supply and the resistance of the inductance as found by the first method, the value of the inductance corresponding to the particular value of direct current was computed. The circuit used for this method is shown in Figure 24. The values of L as a function of direct current are plotted in Figure 25. It is to be noted that the values determined by method one check quite closely with the values found by method two.

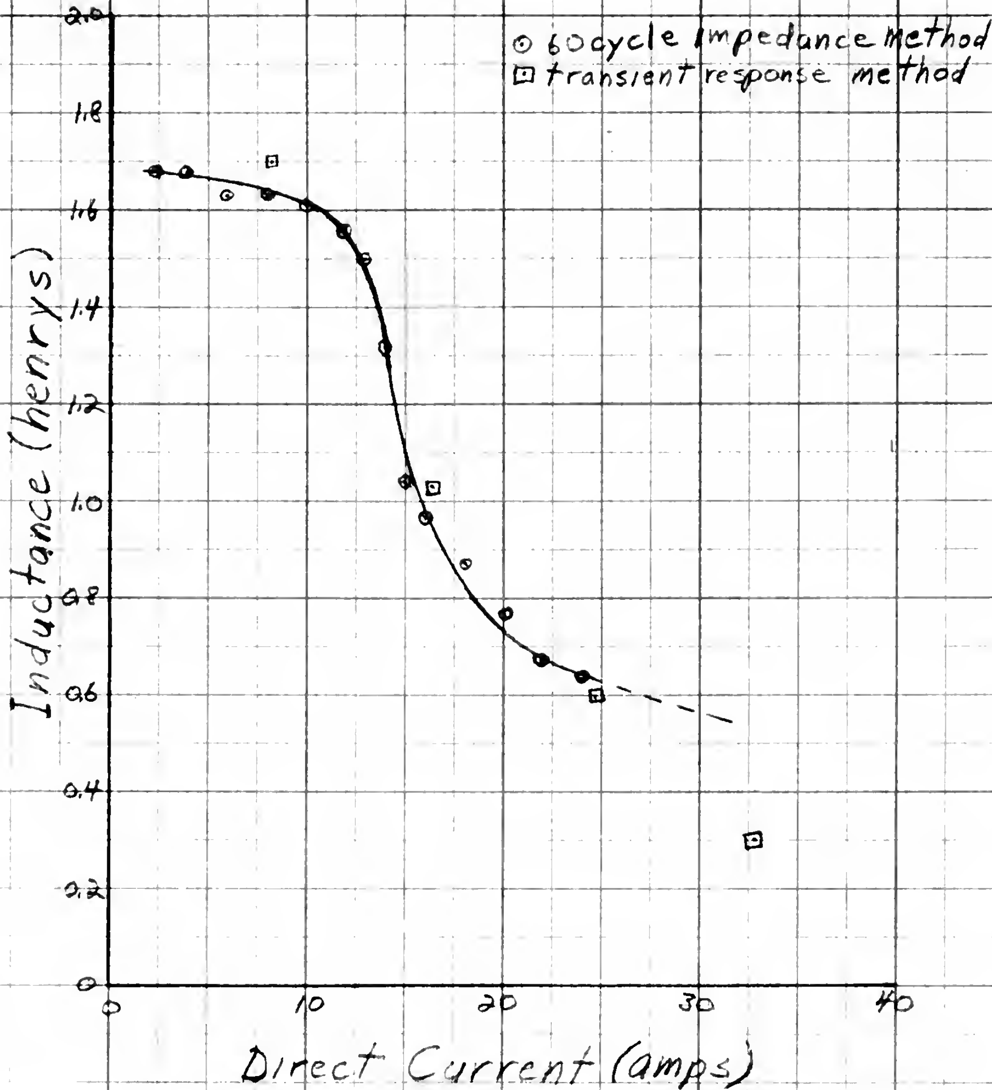


Photograph of the Starting Inductance

Figure 23



Circuit for the Measurement of the
Starting Inductance
Figure 24



Starting Inductance as a Function of Current

Figure 25

APPENDIX D

COMPUTER SOLUTIONS OF MOTOR STARTING EQUATIONS

This appendix consists of the graphical solutions obtained by solving the motor starting equations for the 7.5 horsepower motor described in Appendix A on the analog computer. Each figure gives the solution for current (i) and speed (ω). All parameters were held constant with the exception of the following which were varied over the ranges shown:

L - Inductance External to Motor
(1 to 5 henrys)

J - Inertia of all Rotating Parts
(0.204 to 0.818 slug-ft^2)

T_c- Load Torque which is constant
(0.0 to 44.76 lbs-ft)

T_s- Load Torque Constant for Load Torque which is proportional to speed
(0.0 to 0.621 lbs-ft/rad/sec)

A tabulation of these solutions appears in Figures 9 and 10 in Chapter III. The results obtained from these solutions are summarized in Figures 11 through 18 in Chapter III.

[illegible]

This [illegible]

solving [illegible]

describe [illegible]

the solution [illegible]

held constant [illegible]

over the range [illegible]:

L – [illegible]

L – [illegible]

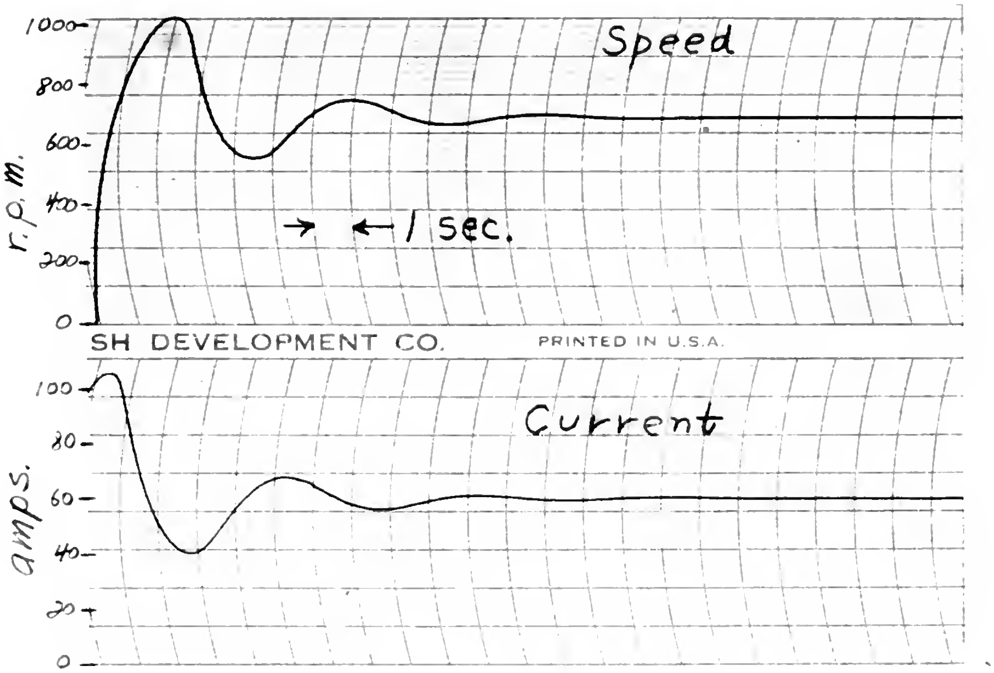
T_s – [illegible]

T_s – [illegible]

A [illegible]

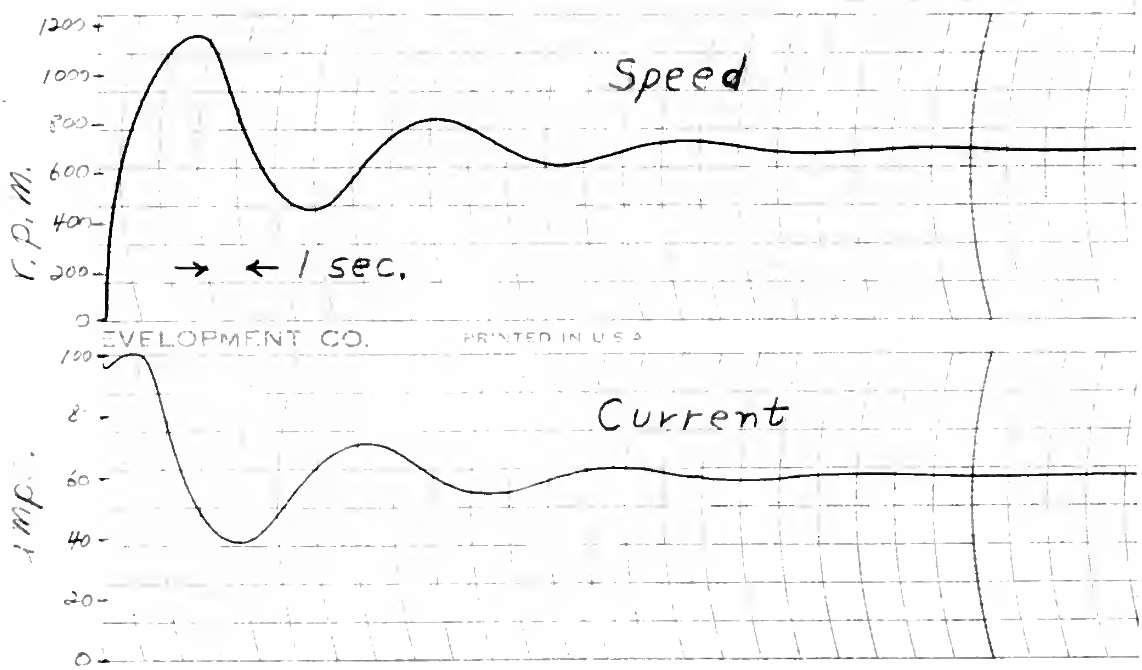
Chapter [illegible]

in Figure [illegible].



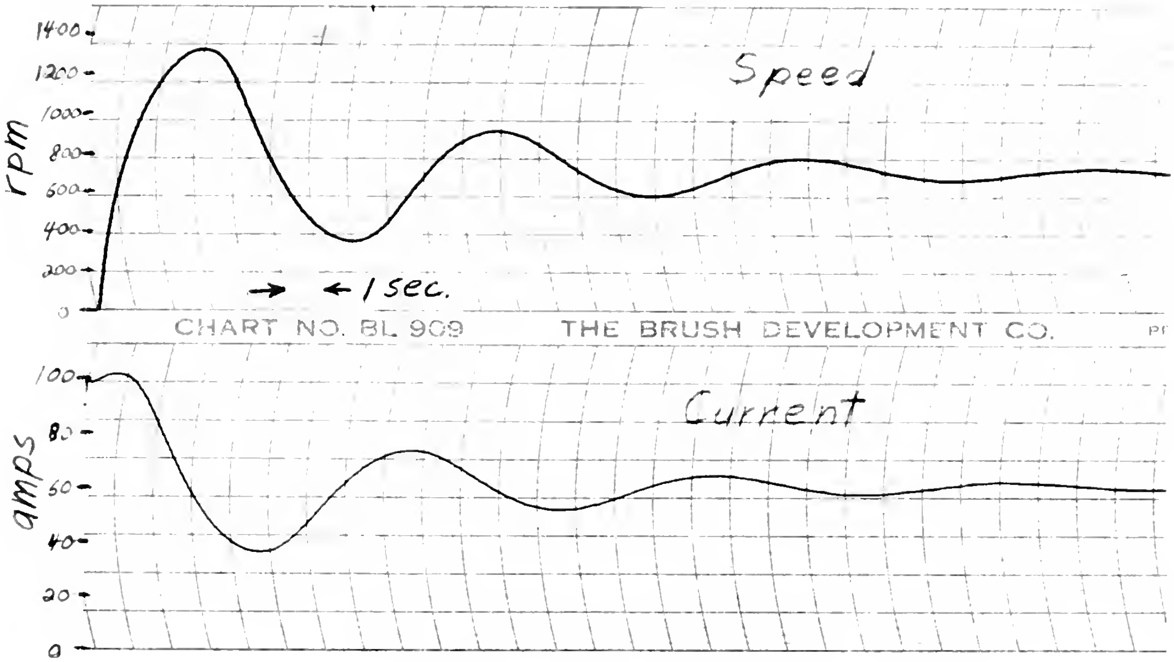
$L = 1$ $J = 0.409$ $T_c = 44.76$ $T_s = 0$

Figure 26



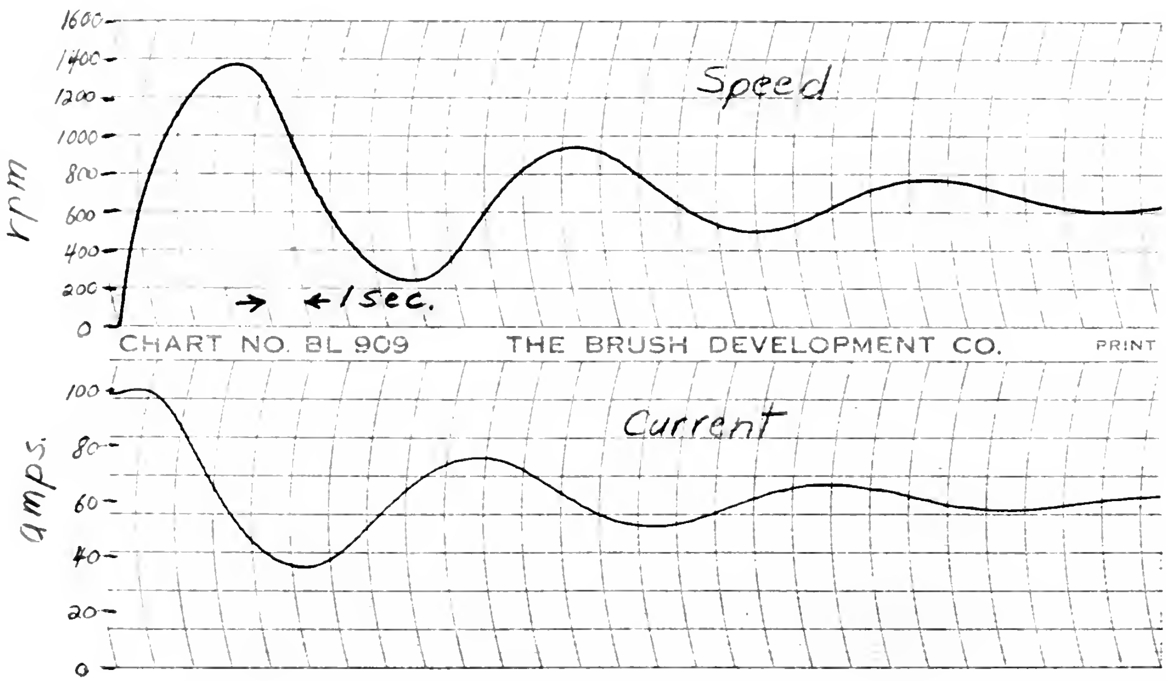
$L = 2$ $J = 0.409$ $T_c = 44.76$ $T_s = 0$

Figure 27



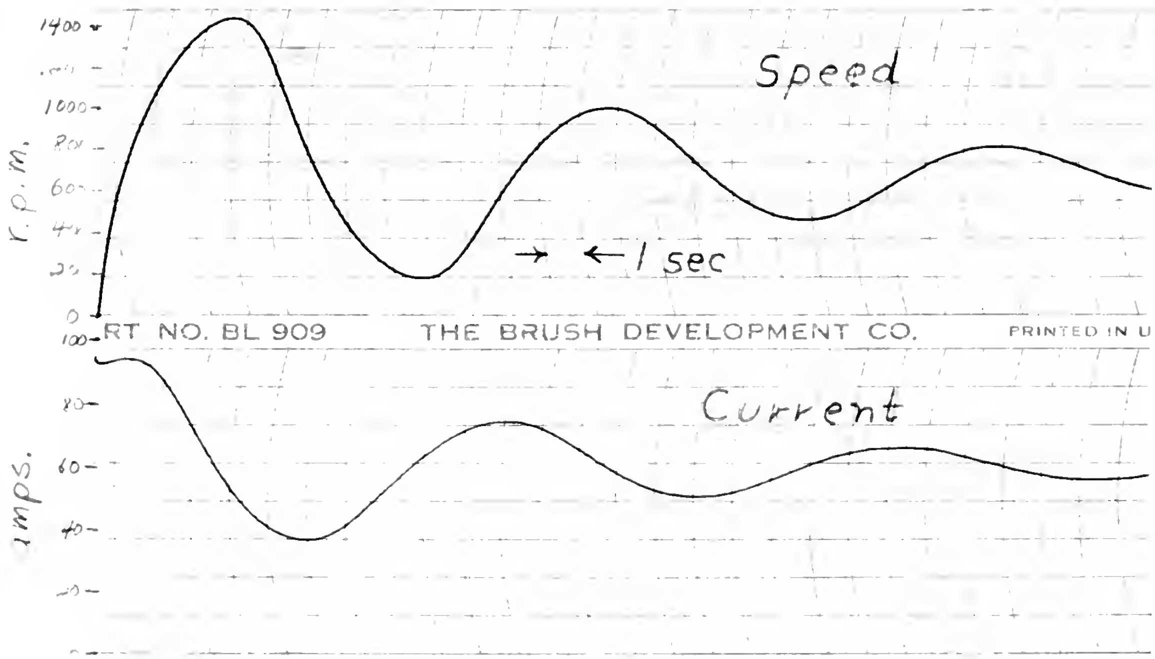
$L = 3 \quad J = 0.409 \quad T_c = 44.76 \quad T_s = 0$

Figure 28



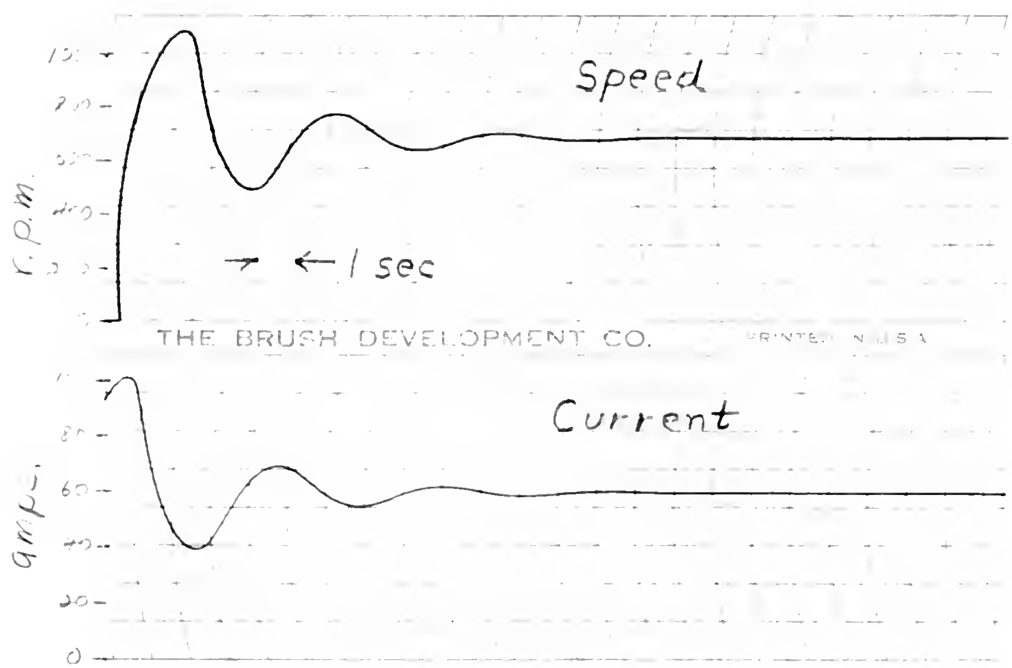
$L = 4 \quad J = 0.409 \quad T_c = 44.76 \quad T_s = 0$

Figure 29



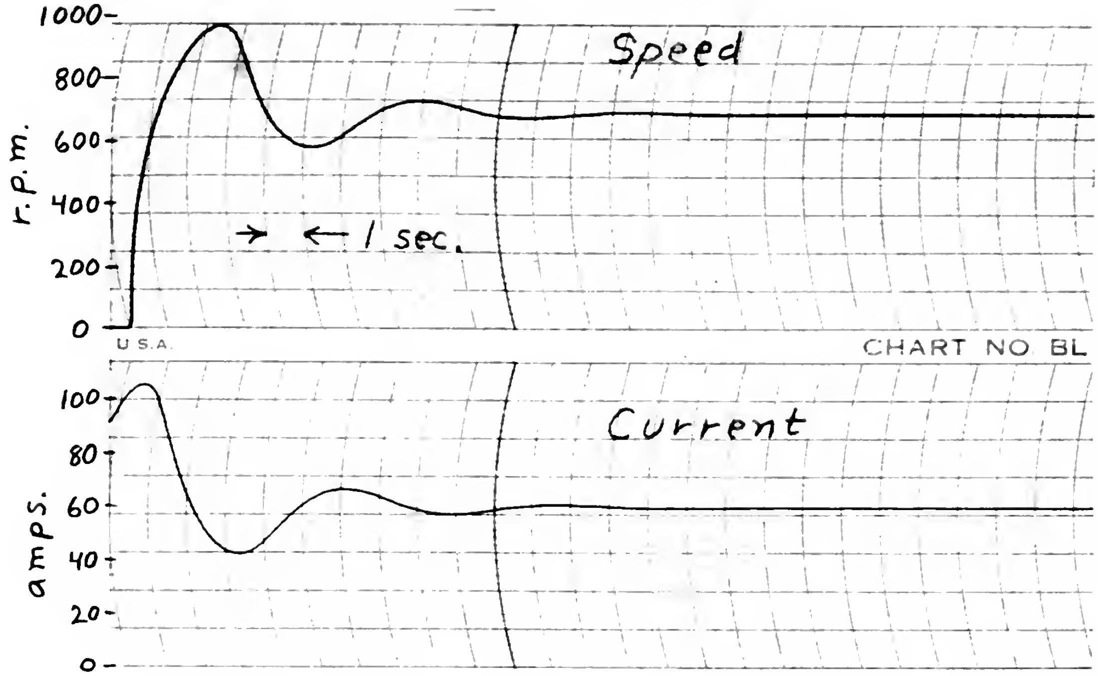
$L = 5 \quad J = 0.409 \quad T_c = 44.76 \quad T_s = 0$

Figure 30



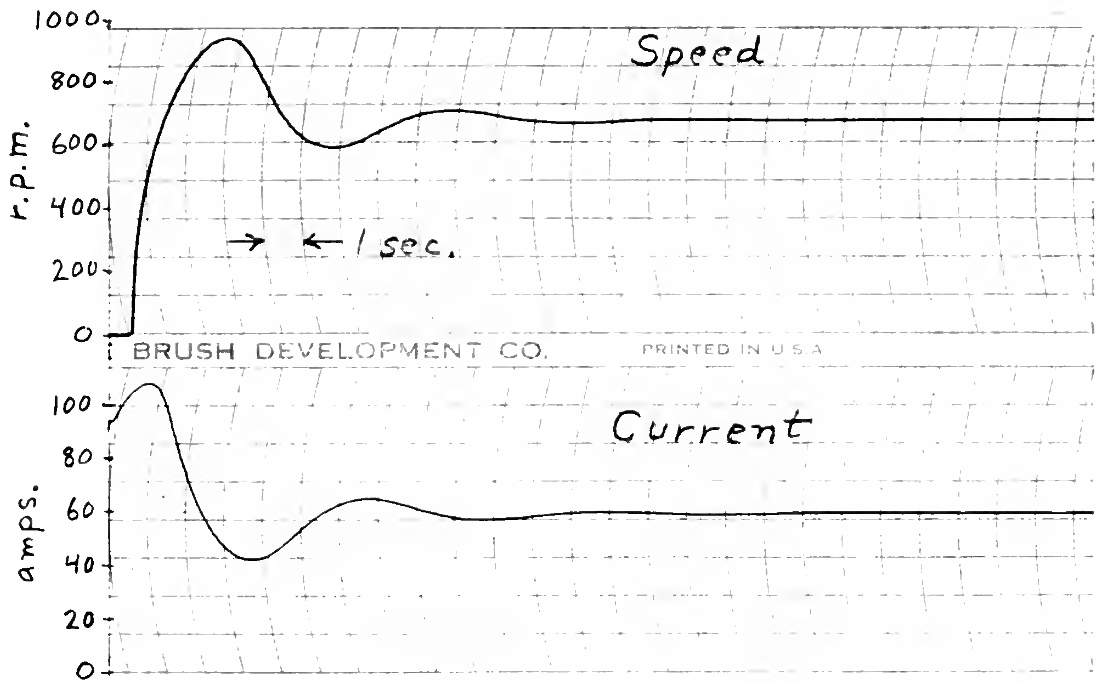
$L = 1 \quad J = 0.300 \quad T_c = 44\,76 \quad T_s = 0$

Figure 31



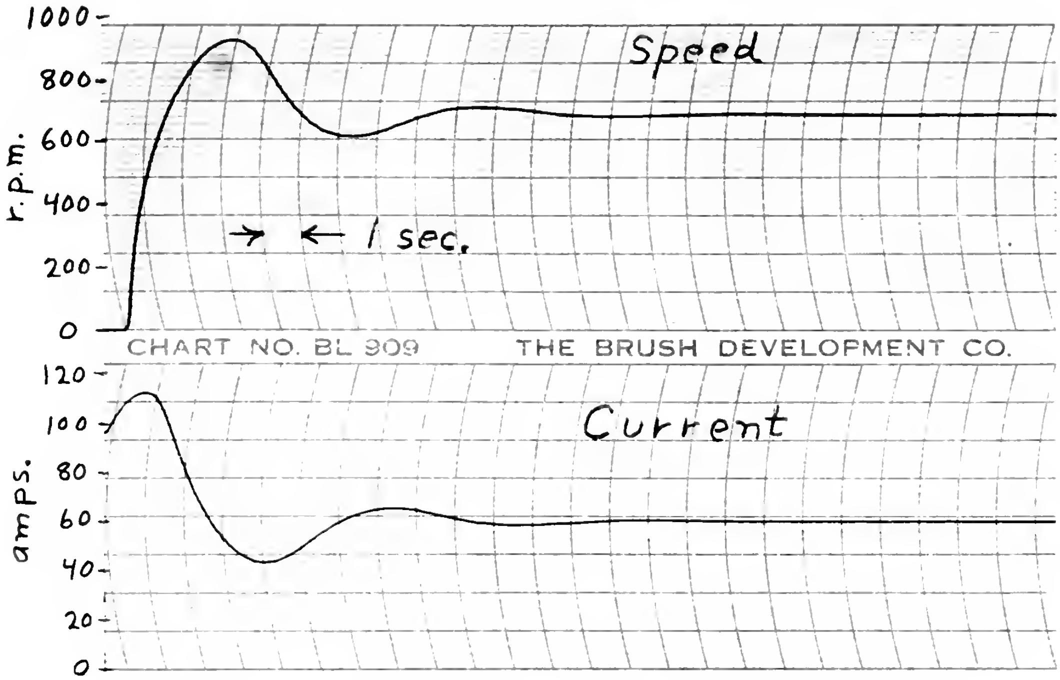
L = 1 J = 0.500 T_c = 44.76 T_s = 0

Figure 32



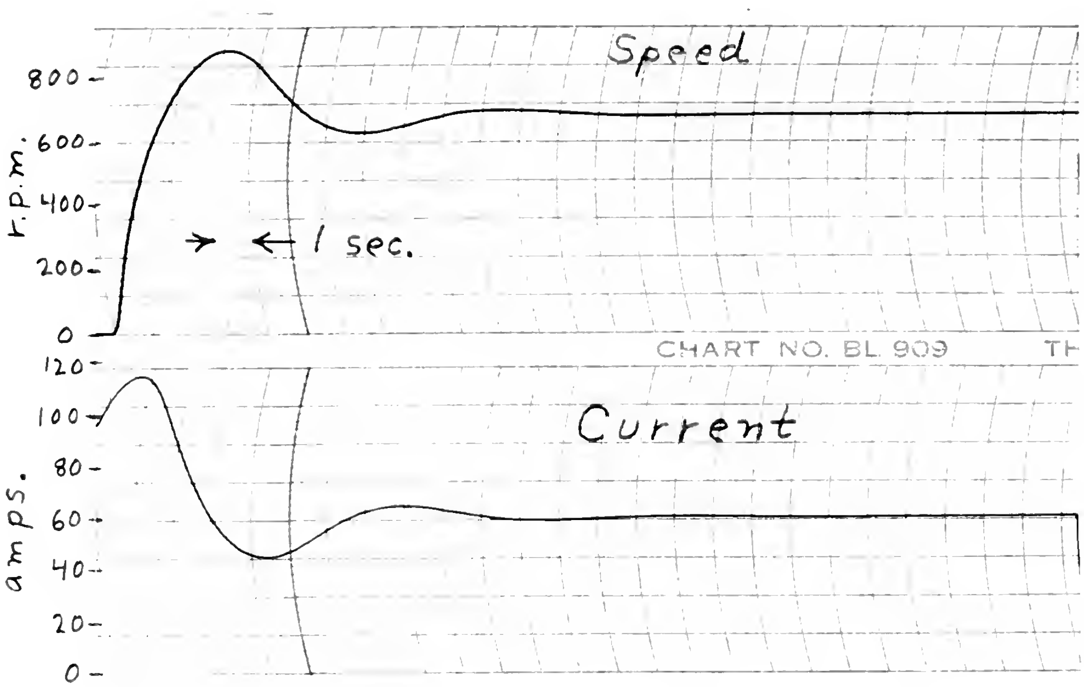
L = 1 J = 0.600 T_c = 44.76 T_s = 0

Figure 33



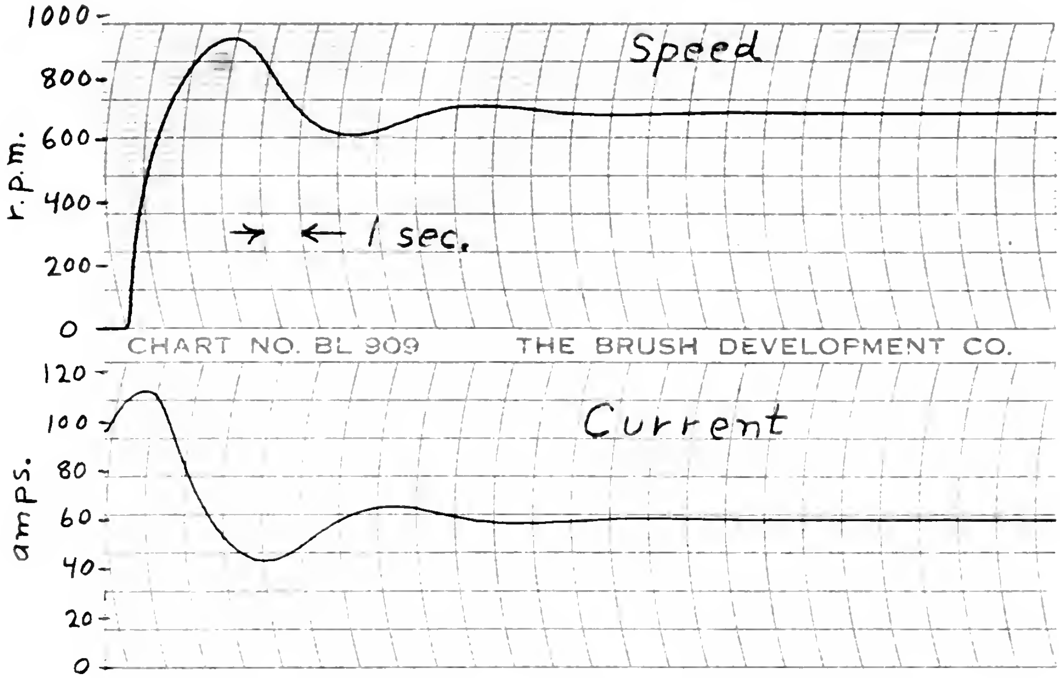
$L = 1 \quad J = 0.700 \quad T_c = 44.76 \quad T_s = 0$

Figure 34



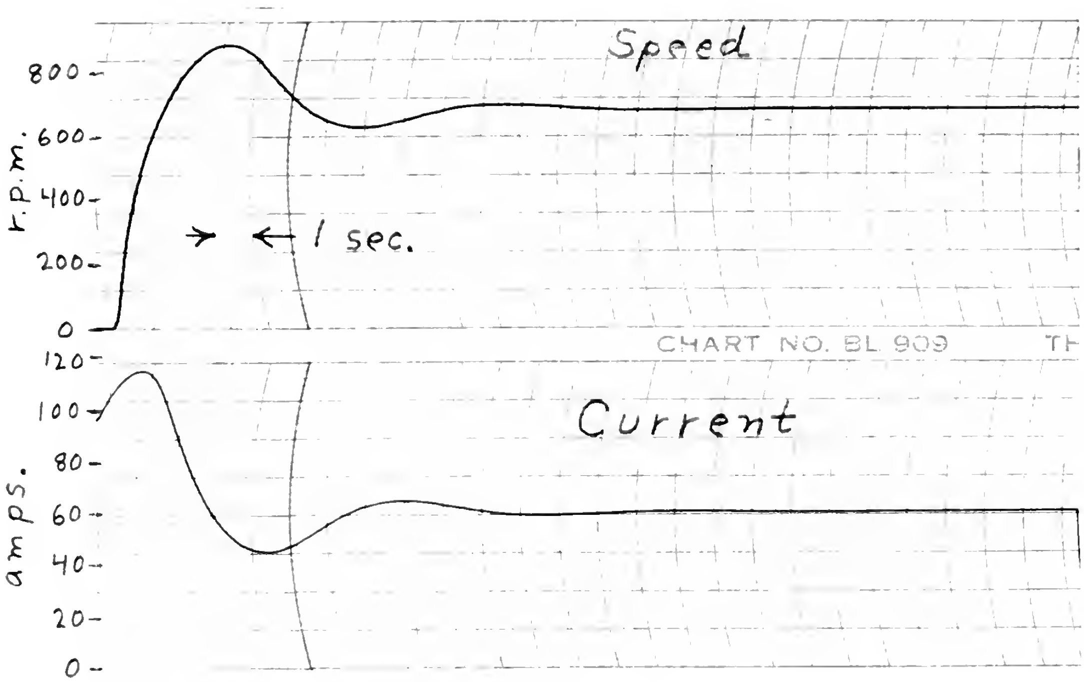
$L = 1 \quad J = 0.818 \quad T_c = 44.76 \quad T_s = 0$

Figure 35



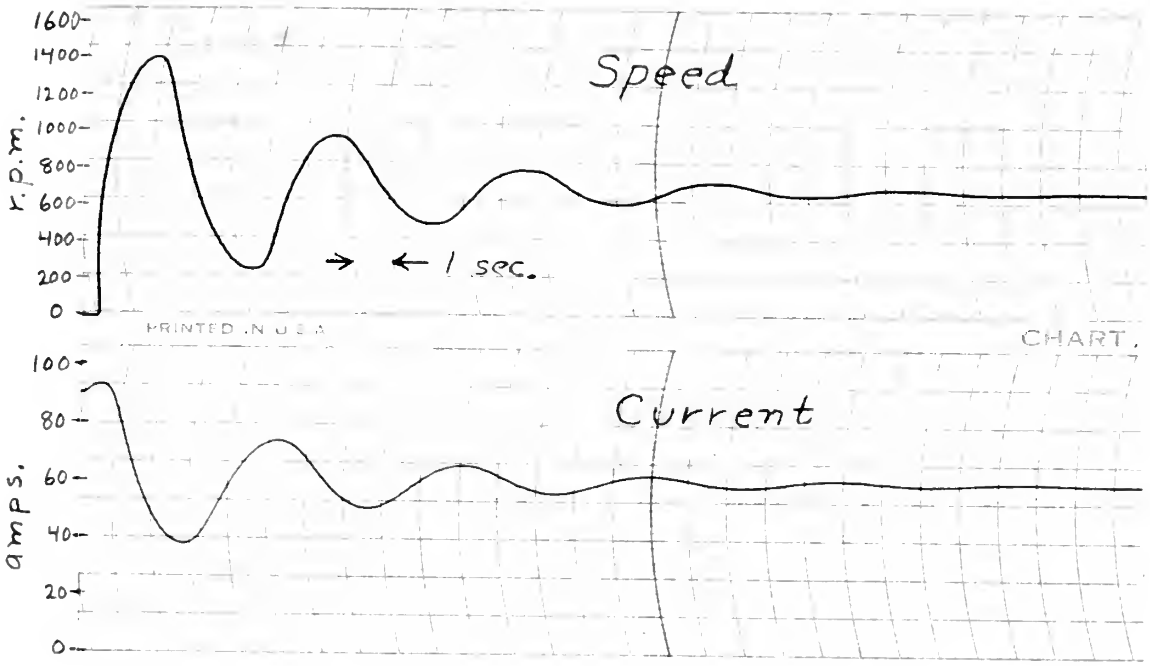
L = 1 J = 0.700 T_c = 44.76 T_s = 0

Figure 34



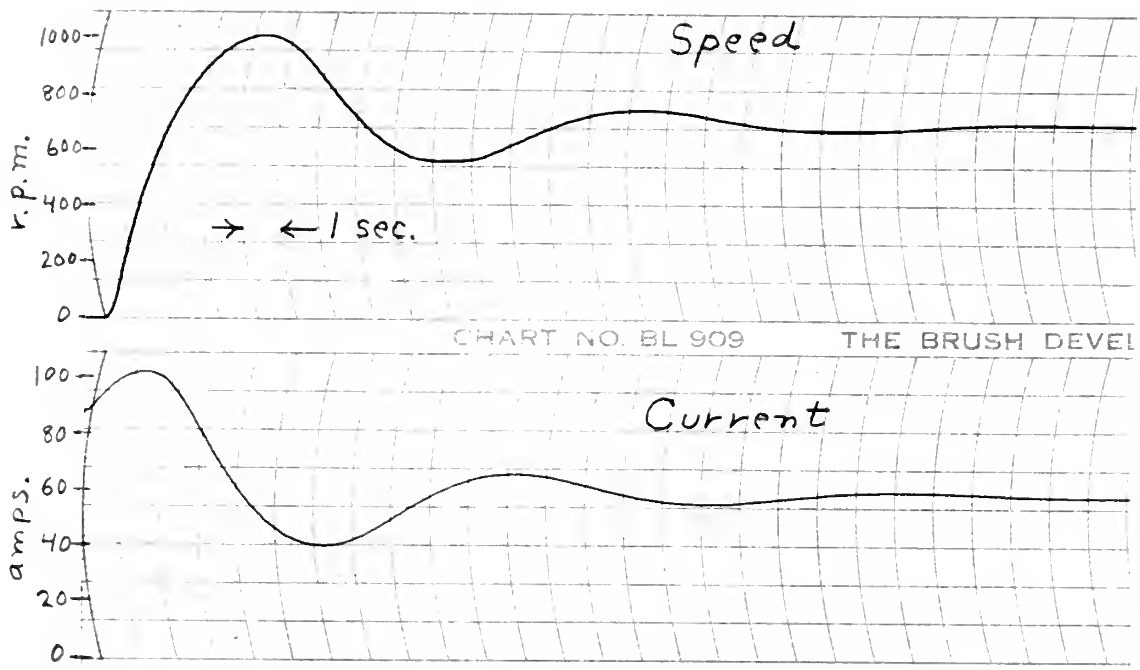
L = 1 J = 0.818 T_c = 44.76 T_s = 0

Figure 35



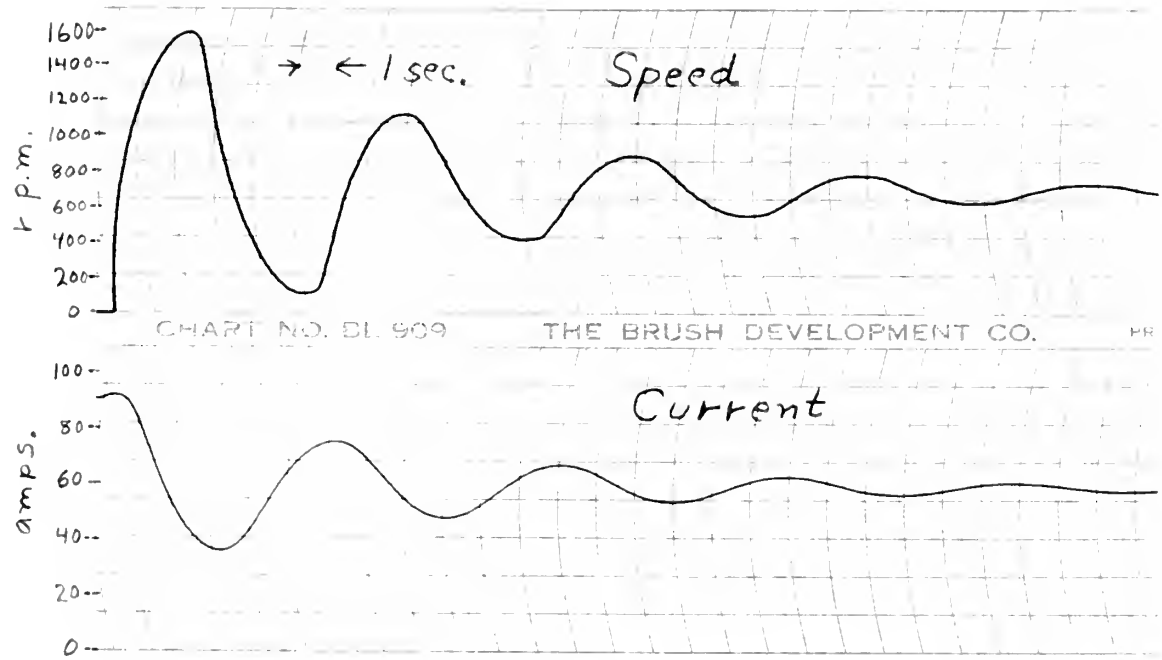
$L = 2$ $J = 0.204$ $T_c = 44.76$ $T_s = 0$

Figure 36



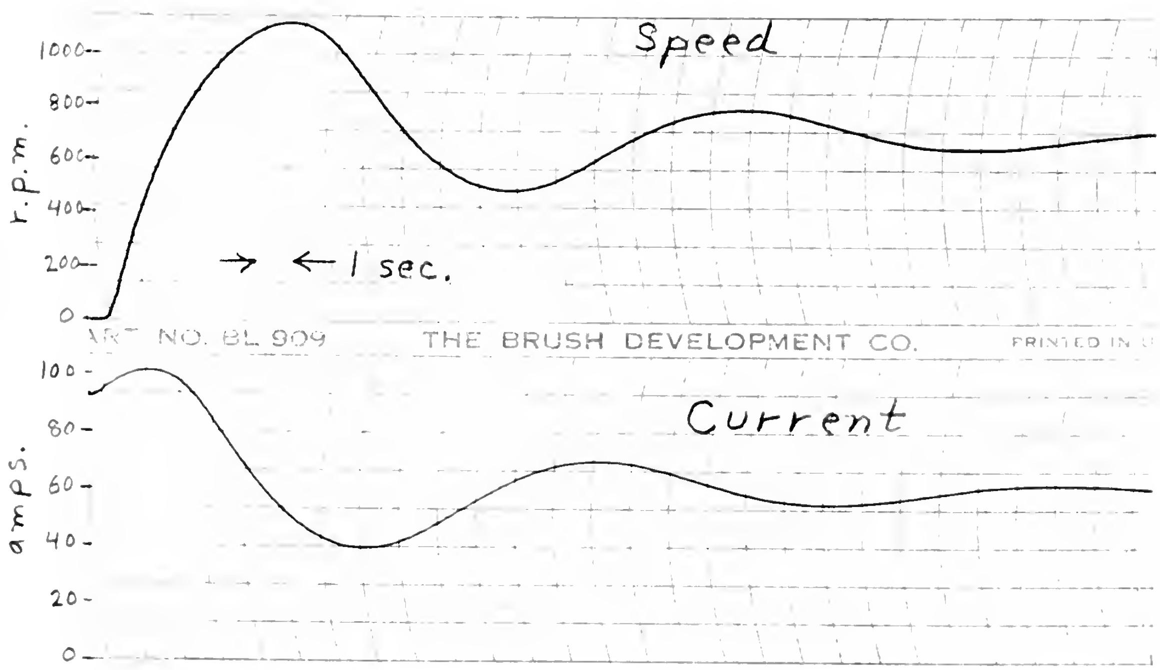
$L = 2$ $J = 0.818$ $T_c = 44.76$ $T_s = 0$

Figure 37



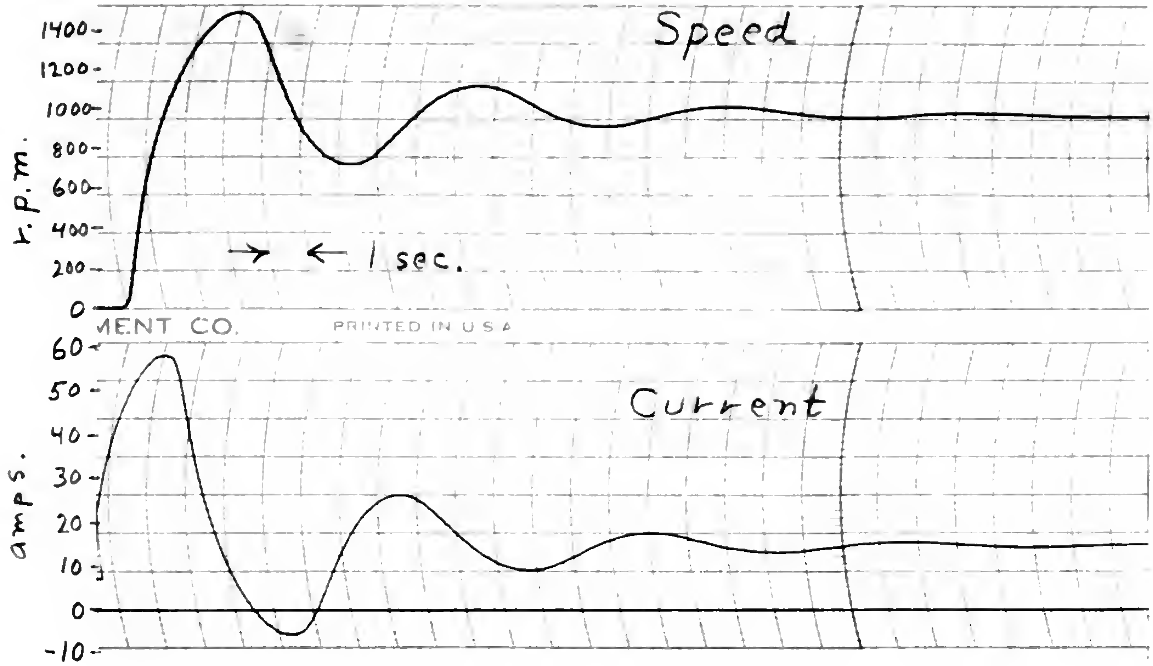
$L = 3$ $J = 0.204$ $T_c = 44.76$ $T_s = 0$

Figure 38



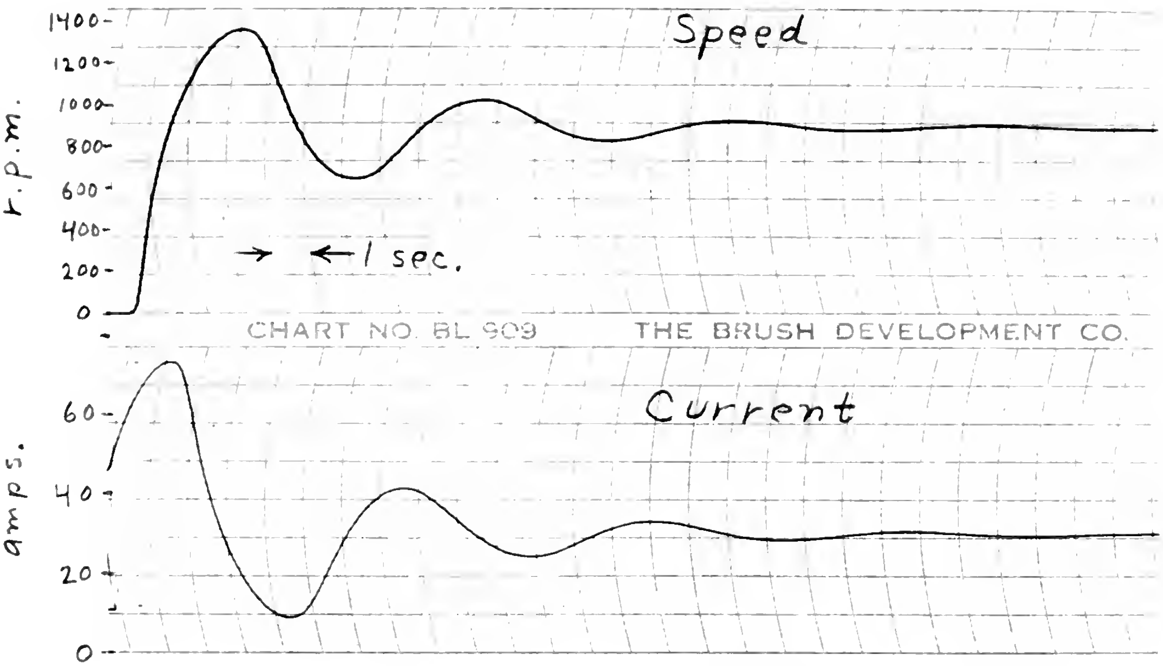
$L = 3$ $J = 0.818$ $T_c = 44.76$ $T_s = 0$

Figure 39



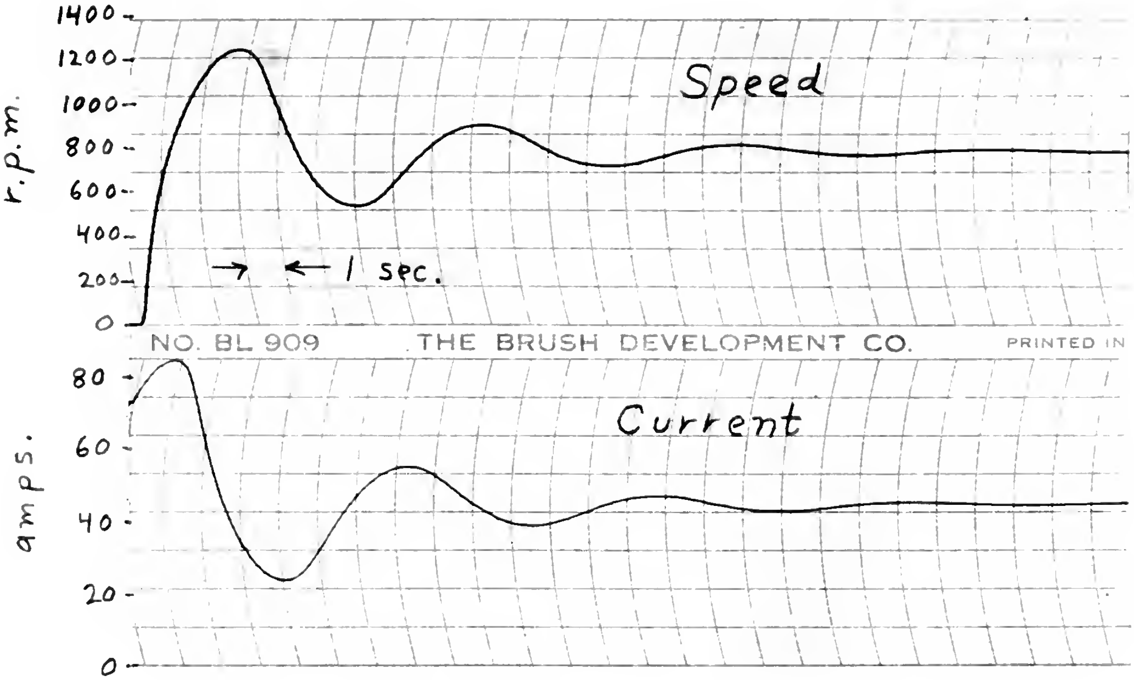
$L = 2$ $J = 0.409$ $T_c = 11.19$ $T_s = 0$

Figure 40



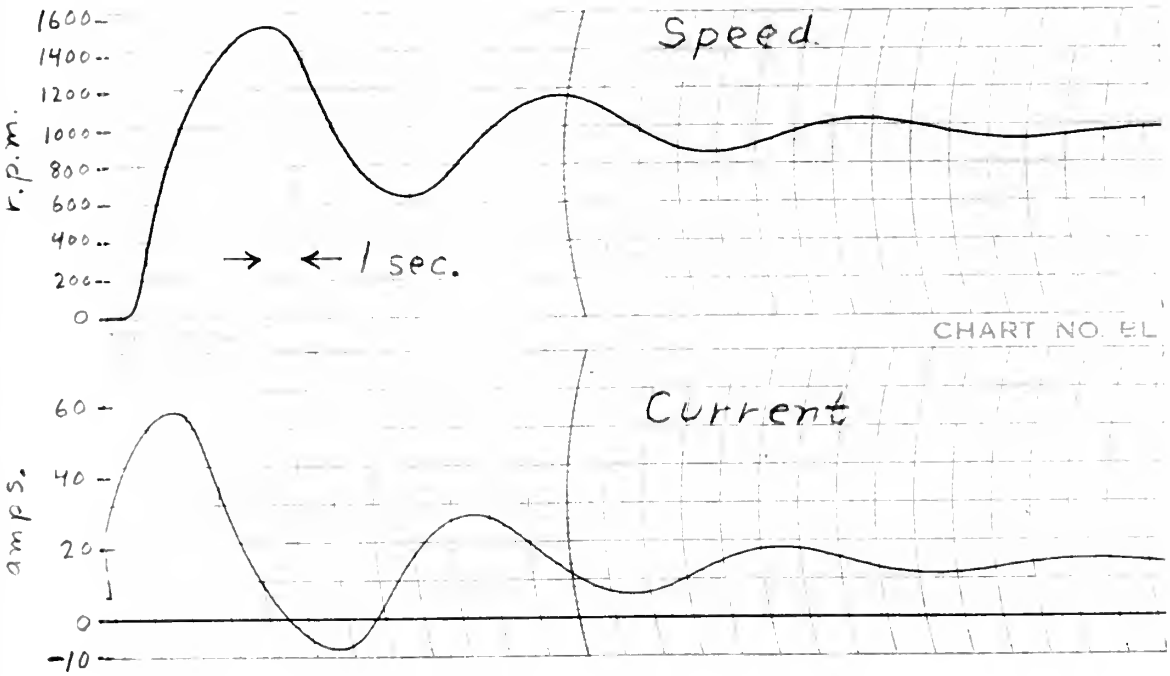
$L = 2$ $J = 0.409$ $T_c = 22.38$ $T_s = 0$

Figure 41



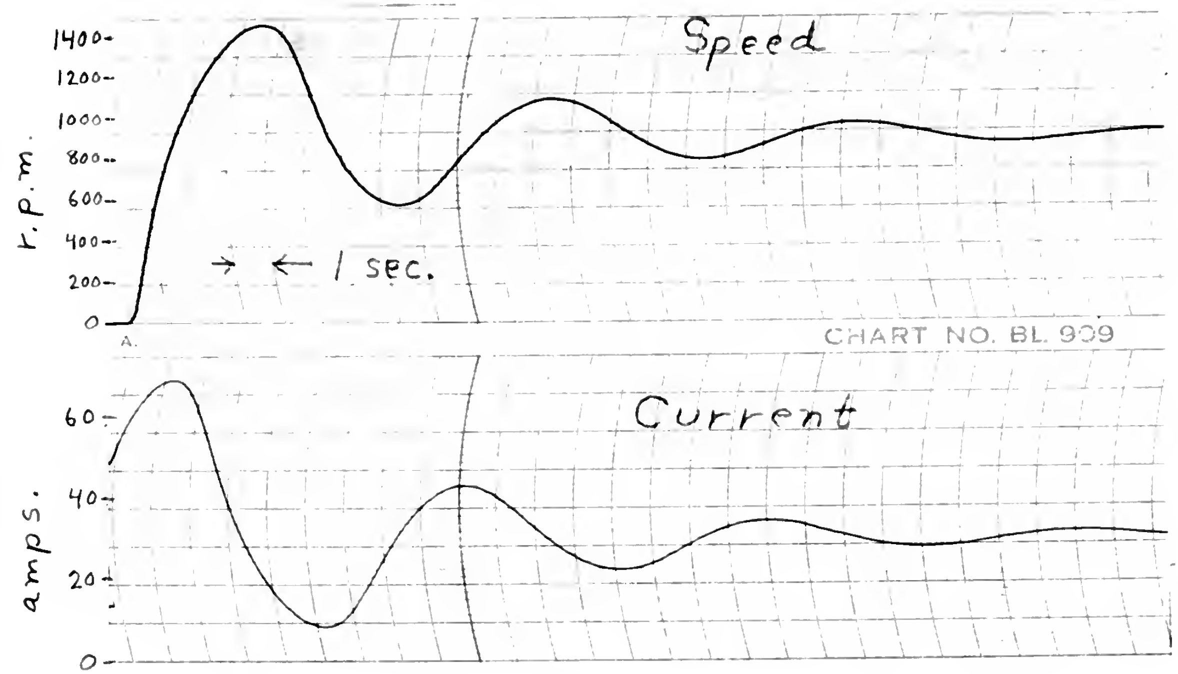
L = 2 J = 0.409 Tc = 33.57 Ts = 0

Figure 42



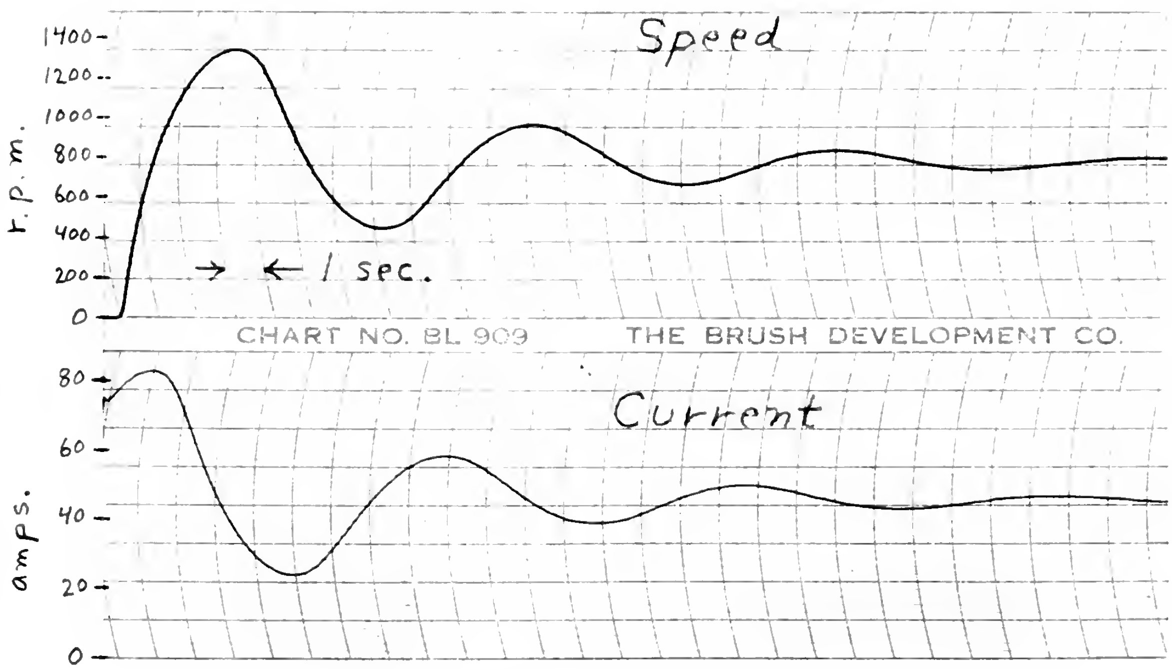
L = 3 J = 0.409 Tc = 11.19 Ts = 0

Figure 43



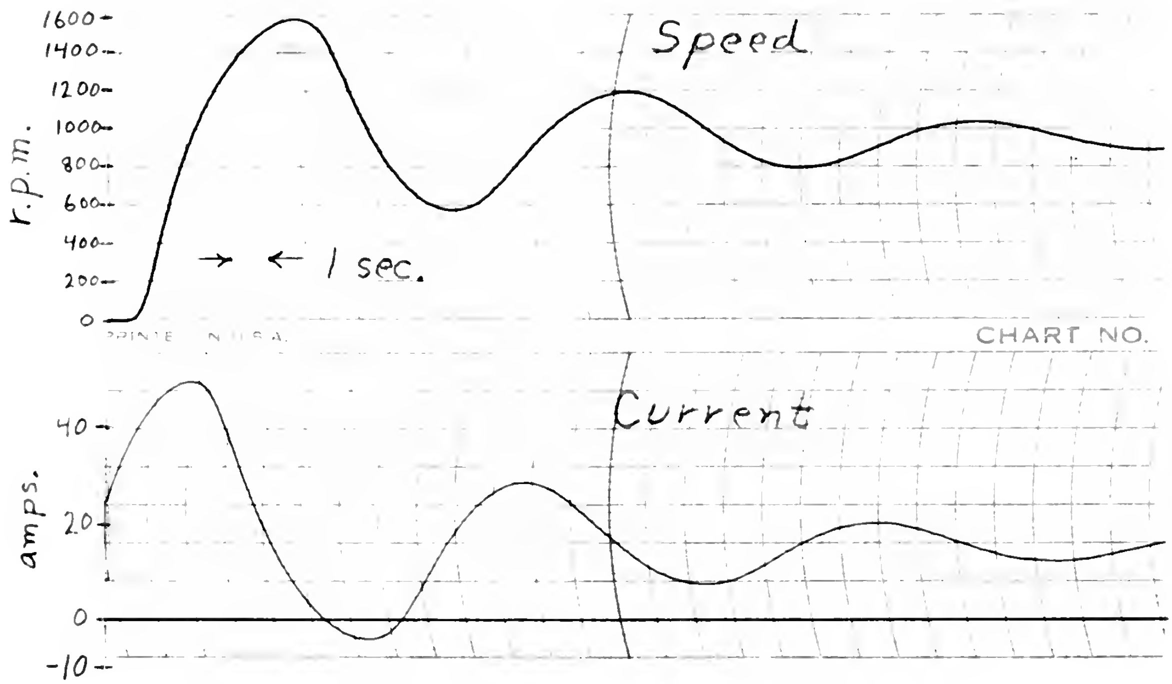
$L = 3$ $J = 0.409$ $T_c = 22.38$ $T_s = 0$

Figure 44



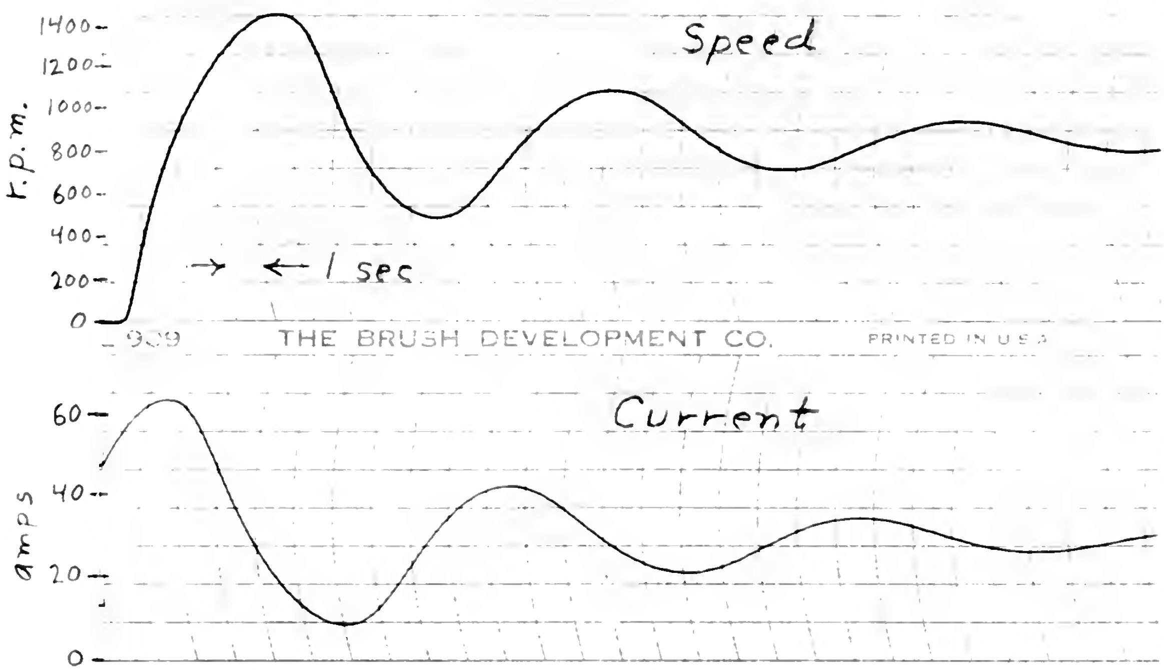
$L = 3$ $J = 0.409$ $T_c = 33.57$ $T_s = 0$

Figure 45



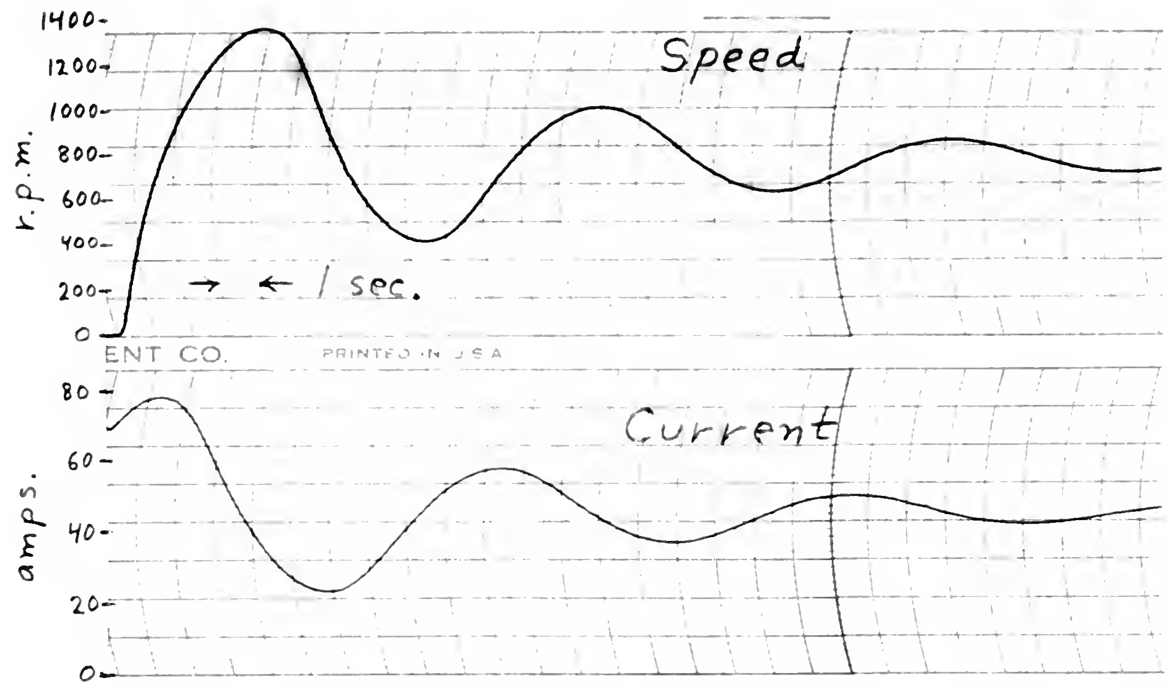
L = 4 J = 0.409 T_c = 11.19 T_s = 0

Figure 46



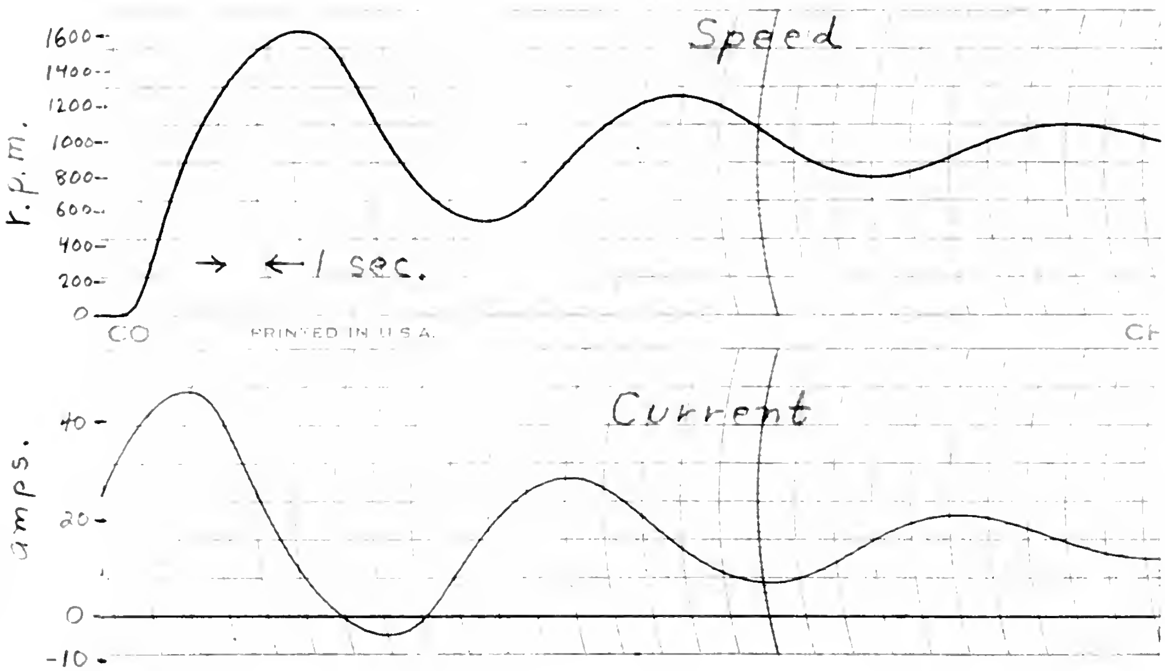
L = 4 J = 0.409 T_c = 22.38 T_s = 0

Figure 47



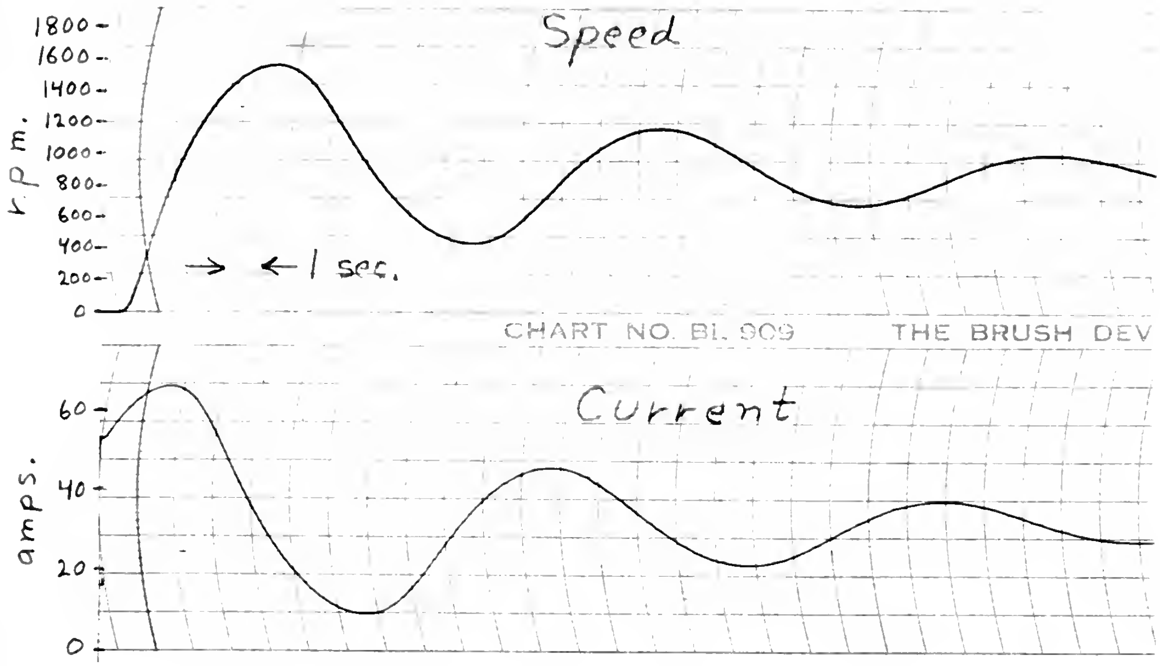
$L = 4$ $J = 0.409$ $T_c = 33.57$ $T_s = 0$

Figure 48



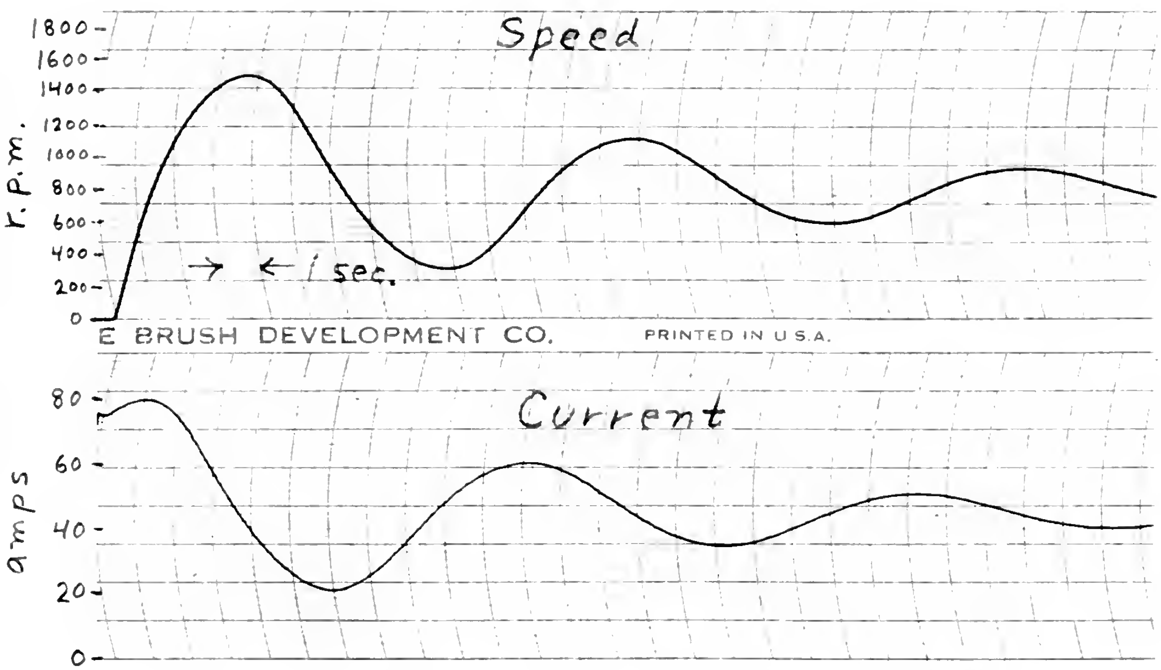
$L = 5$ $J = 0.409$ $T_c = 11.19$ $T_s = 0$

Figure 49



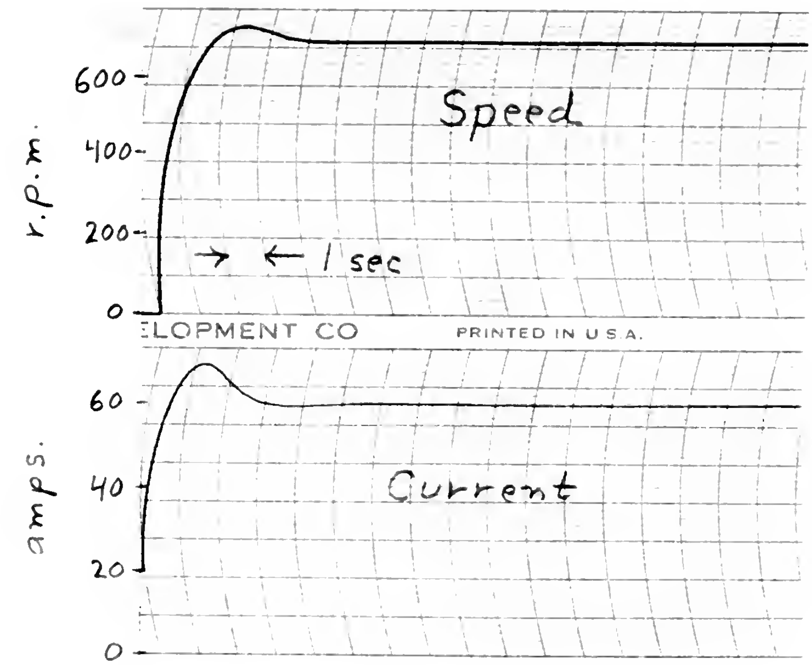
L = 5 J = 0.409 Tc = 22.38 Ts = 0

Figure 50



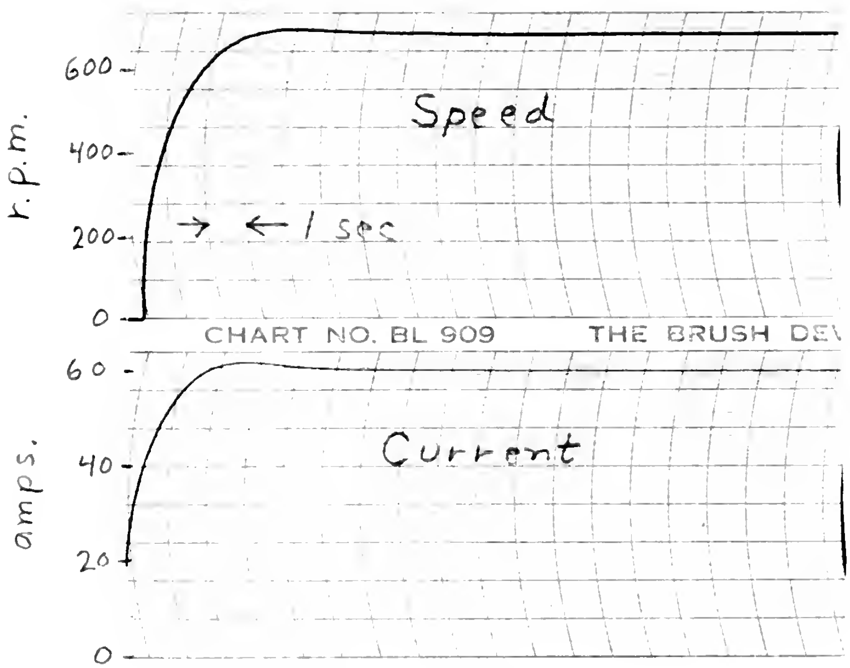
L = 5 J = 0.409 Tc = 33.57 Ts = 0

Figure 51



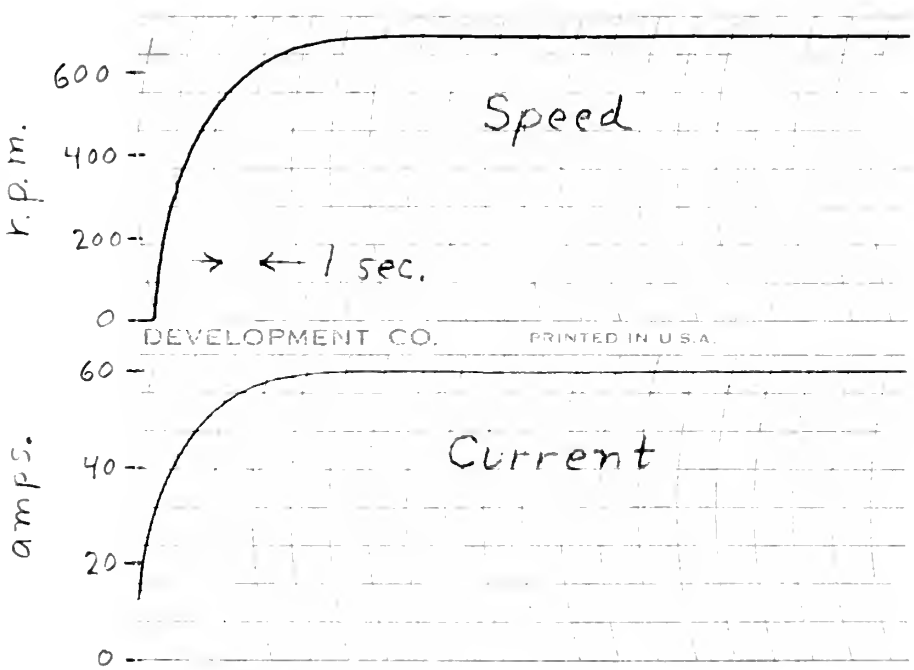
L = 1 J = 0.409 Tc = 0 Ts = 0.621

Figure 52



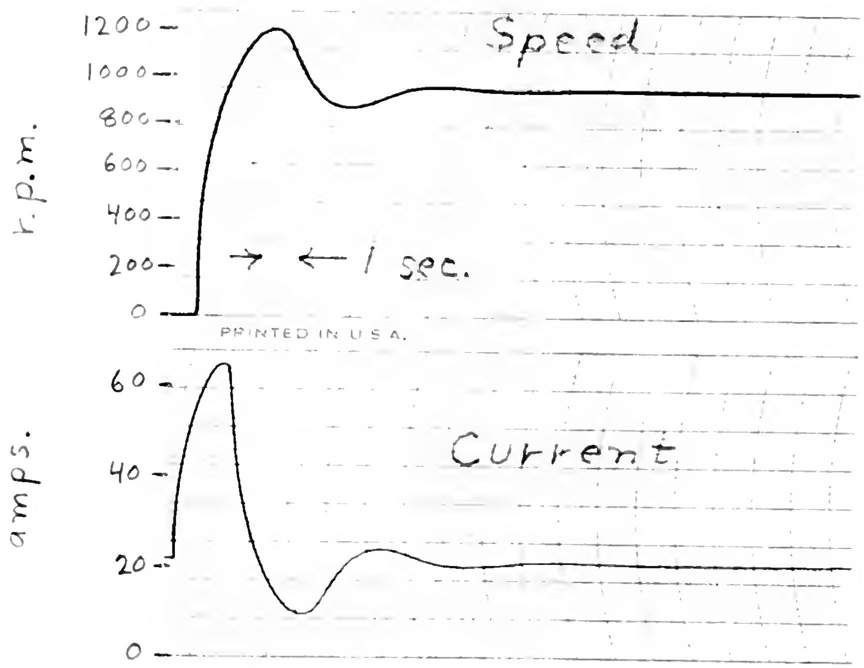
L = 2 J = 0.409 Tc = 0 Ts = 0.621

Figure 53



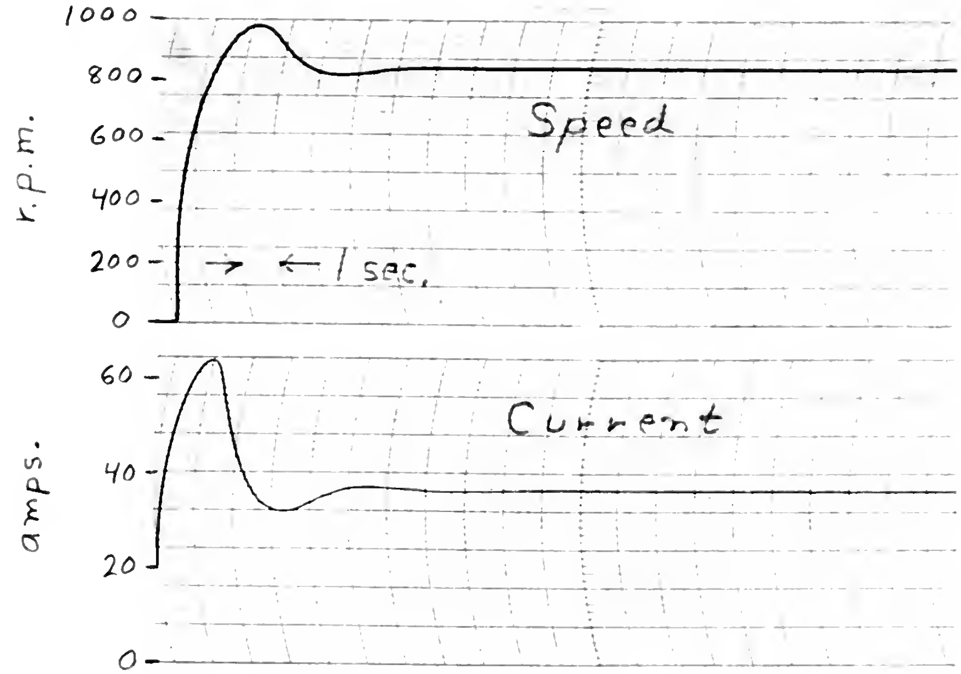
$L = 3$ $J = 0.409$ $T_c = 0$ $T_s = 0.621$

Figure 54



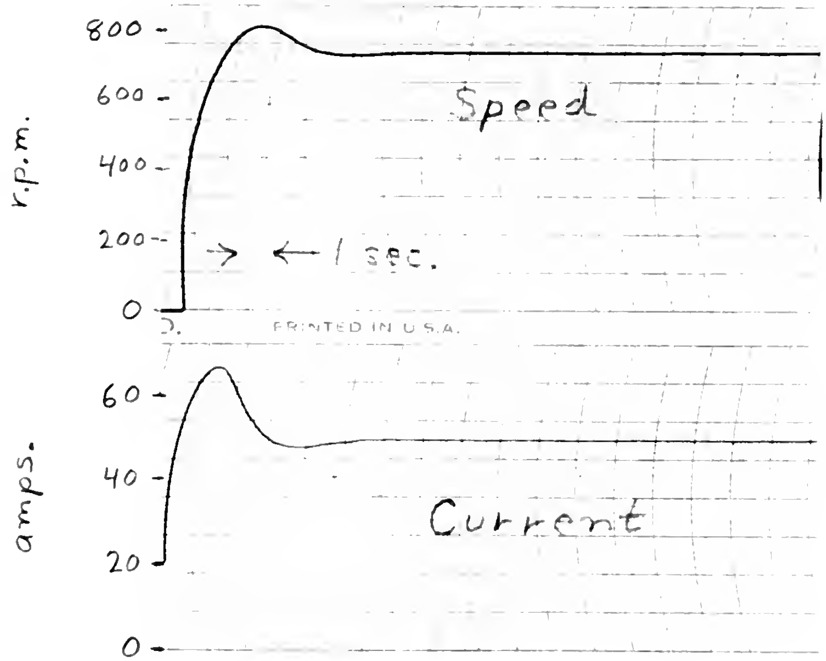
$L = 1$ $J = 0.409$ $T_c = 0$ $T_s = 0.155$

Figure 55



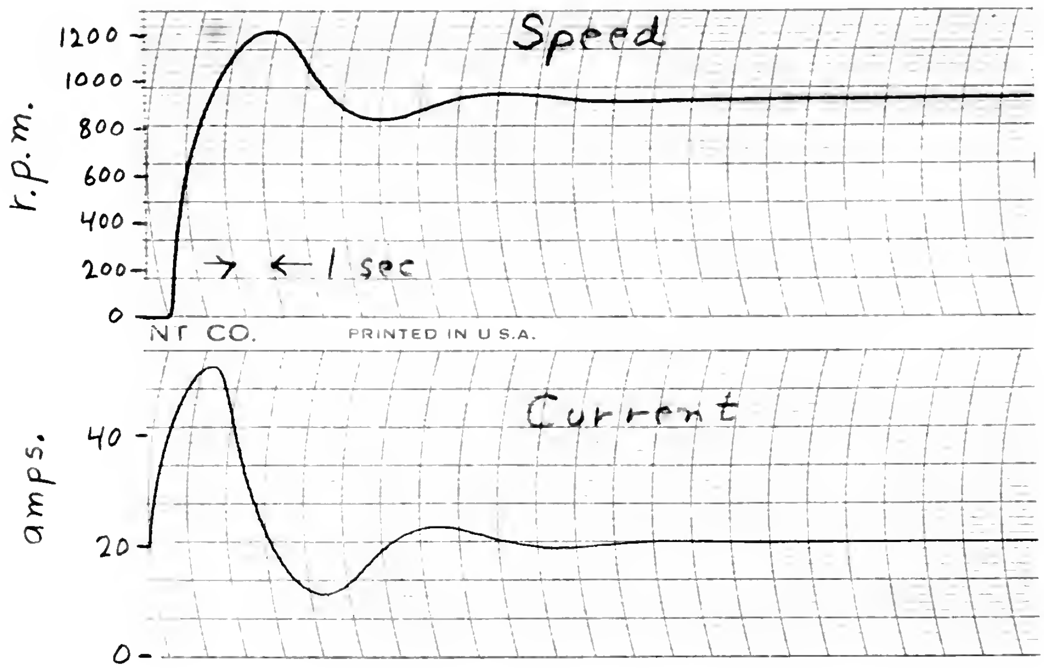
L = 1 J = 0.409 Tc = 0 Ts = 0.310

Figure 56



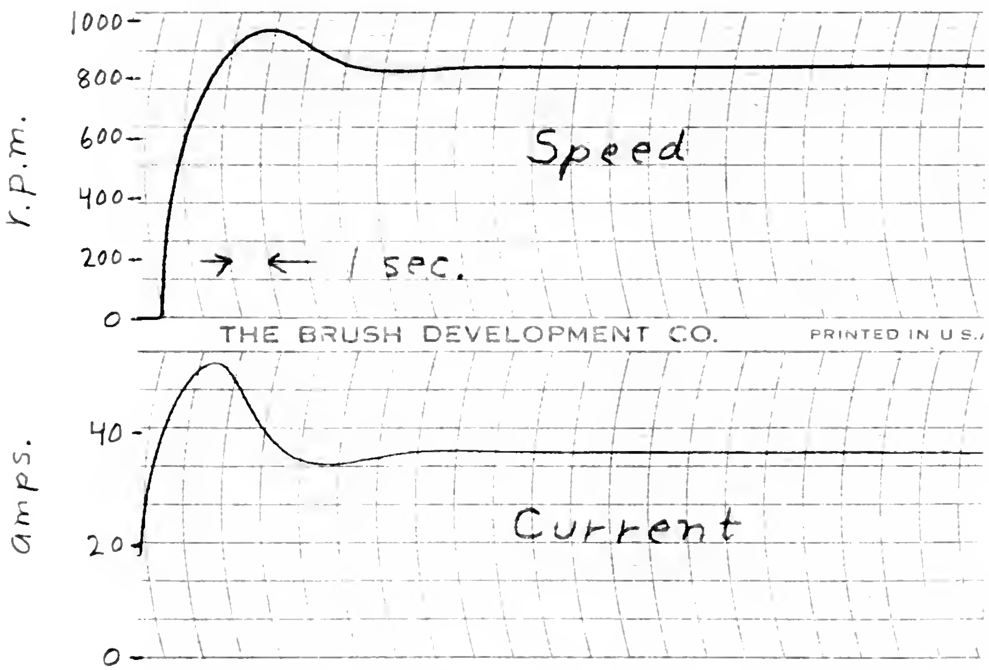
L = 1 J = 0.409 Tc = 0 Ts = 0.466

Figure 57



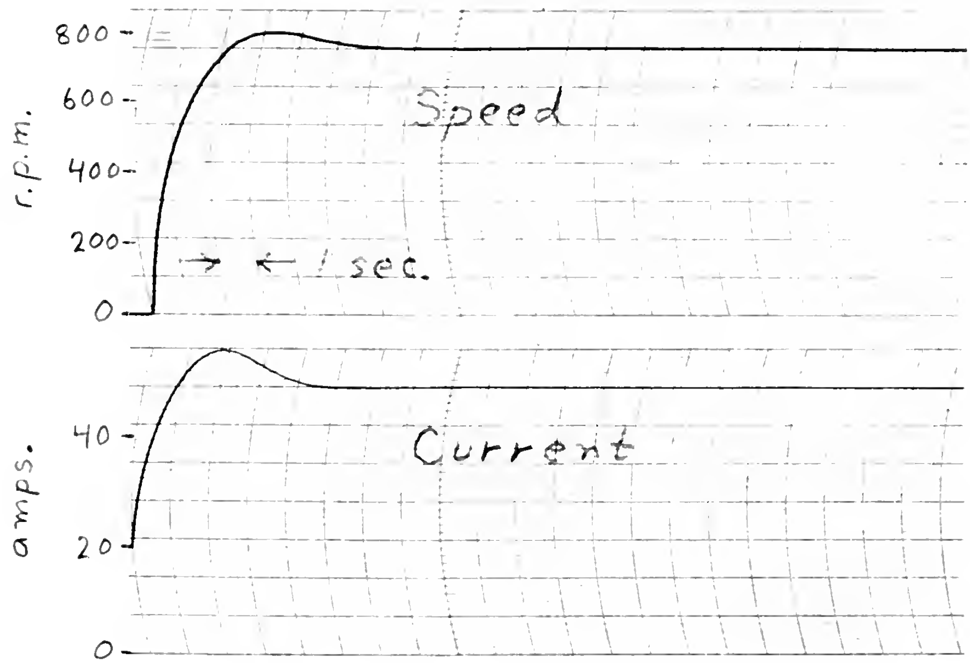
$L = 2$ $J = 0.409$ $T_c = 0$ $T_s = 0.155$

Figure 58



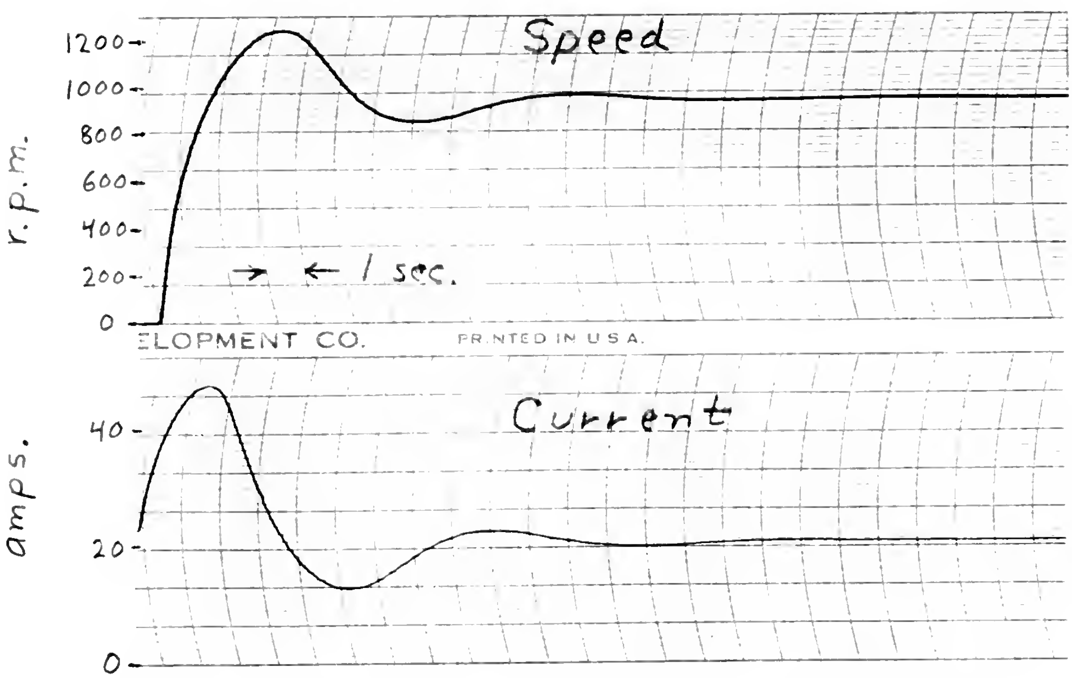
$L = 2$ $J = 0.409$ $T_c = 0$ $T_s = 0.310$

Figure 59



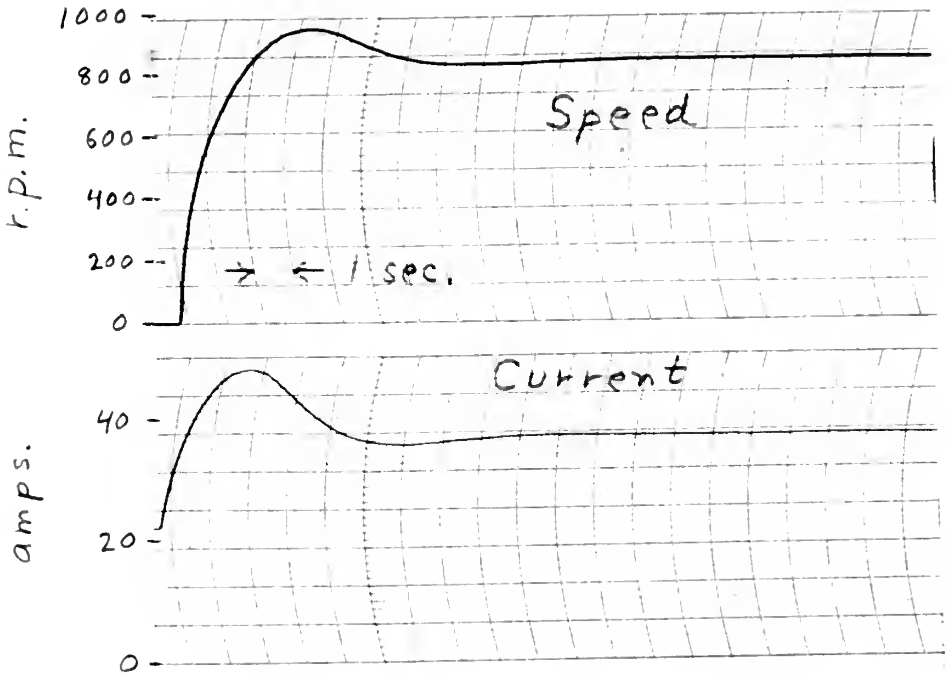
$L = 2$ $J = 0.409$ $T_c = 0$ $T_s = 0.466$

Figure 60



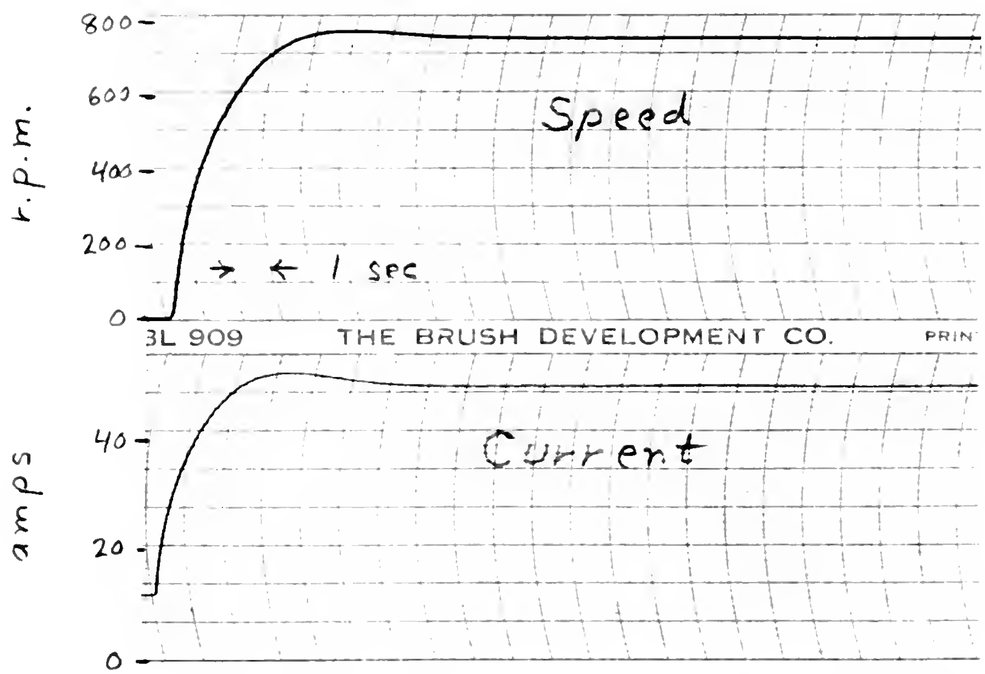
$L = 3$ $J = 0.409$ $T_c = 0$ $T_s = 0.155$

Figure 61



L = 3 J = 0.409 Tc = 0 Ts = 0.310

Figure 62



L = 3 J = 0.409 Tc = 0 Ts = 0.466

Figure 63